

# Getting Down and Dirty: Incorporating Homogeneous Transformations and Robot Kinematics into a Computer Science Robotics Class

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## Abstract

The purpose of this paper is to encourage those instructors who teach Robotics from a Computer Science perspective to include a taste of homogeneous transforms and robot kinematics in their classes. I include some supplemental resources that can help in this endeavor, and some ideas for how to incorporate these topics into mobile robot projects.

## Introduction

The world of Robotics Education often is divided into two camps: the Mechanical Engineers with their emphasis on hardware and low level control, and the Computer Scientists, who tend to focus on AI and autonomy.

Most textbooks reflect this divide. On the Computer Science side, most books do provide a brief introduction to some of the engineering issues in Robotics, but the vast majority leave out even the gentlest introduction to homogeneous transforms and robot kinematics.

The purpose of this paper is to encourage those instructors who teach Robotics from a Computer Science perspective to include a taste of homogeneous transforms and robot kinematics in their classes. I include some supplemental resources that can help in this endeavor, and some ideas for how to incorporate these topics into mobile robot projects.

## Why Include Kinematics in a Computer Science Robotics Course?

One might argue that the primary reason is that many of these Computer Science courses go under the name "Robotics" and that there is a great value in including some coverage of robot manipulators and the mathematics behind them in any Robotics course.

Beyond that, taking some time to study homogeneous transforms as a means to thinking about relative coordinate frames is extremely relevant to those working with autonomous mobile robots. As robots move around in a 3D world students need an understanding of where they are relative to other robots and objects, as well as have the ability to simply transform arbitrary points between different coordinate frames.

## Practical Classroom Issues

### How Much of My Class Time Will it Take?

It takes me roughly two and a half weeks (about six and a half hours) of class time to give a very gentle introduction to this material. Students in my class are Juniors and Seniors majoring in CS (and occasionally Electrical Engineering) and so have a reasonable background in math. There is a Linear Algebra prerequisite for my Robotics course, but the only material that I require from that prerequisite is some basic matrix manipulation skills. I do require that they do some of their written homework using Mathematica, which most (but not all) have seen before. It takes a very short time to demonstrate the minimal Mathematica skills necessary to create functions and multiply matrices, and those students who do not have previous Mathematica experience do not find it too difficult.

### What Topics Should be Covered?

- **Coordinate frames:** I begin with a quick review of coordinate frames, as some of the students have not seen the concept of right handed coordinate systems before.
- **3-D drawing techniques:** This is more for self defense than anything else. In the semesters before I explicitly taught some 3-D drawing skills, some students turned in incomprehensible homeworks.
- **Homogeneous Coordinates and Homogeneous Transformations:** This is the heart of the material. We discuss how to represent Rotation and Translation Operators as matrices, and use these matrices to

transform points between coordinate frames. We also cover how to determine the relationship between two different coordinate frames.

- **Forward Kinematics:** I feel it is important to introduce students to robot manipulators, and so I do take some time to present forward kinematics to my class.
- **Using Mathematica:** I ask my students to do written homework on this material, and insist that they use Mathematica. Teaching them the minimal Mathematica skills they need takes very little time.

### What References are Available?

I learned this material from a wonderful Tech Report by Al Kelly [2]. It's extremely well written and comprehensive, but somewhat concise. I used the first few sections of his paper in my class the first two times I taught the course, and it proved to be a bit too intense for my students.

I have subsequently adapted his material into my own document [1] that provides a much gentler introduction to the subject. I have used variations of this document in three different sections of my Robotics class and at this point it works quite well. Both the document itself and slides to accompany the document are available online.

### Ideas for written assignments

I assign two written homeworks for this portion of my class.

In the first assignment, I ask students to compute the homogeneous transformation matrices to transform points between various coordinate frames. In this assignment I also require my students to do a significant amount of graphing just to ensure that they have that basic skill. The second assignment is a basic forward kinematics problem for a 3 joint manipulator.

The two assignments that I gave in my most recent Robotics class can be found at:  
<http://elvis.rowan.edu/~kay/classes/robotics/hw/>

### Ideas for lab assignments

Historically I have treated the first two and a half weeks of my Robotics class as fairly independent from the rest of the semester. We make a big jump from homogeneous transformations, kinematics, and manipulators straight over to AI Robotics without much of a transition. In the future I hope to experiment with incorporating some of the early material into later lab assignments.

My class does not have any meeting time in the lab, however I do assign projects with physical robots to the students to work on outside of class. The lack of a formal time to meet in the lab is somewhat limiting.

The assignments that follow are works in progress. The students have not attempted them yet and so I can not provide any insight at the present time as to how effective they will be. For the purposes of my class, some of the vision and angle detection in these assignments may need to be simulated.

- **Robot Rescue:** This is a simple "multiple coordinate frame" problem. Two mobile robots are hunting for an object to be "rescued." The first one to discover the object reports its own position (in some world coordinate frame) as well as the object's position (in its own coordinate frame). The second robot needs to join the first in order to rescue the object.
- **Triangulation:** In a related problem, two robots both locate an object each using a monocular image sensor, and use triangulation techniques and coordinate frame transformations to determine the location of the object.
- **Triangulation with Hills:** In the triangulation problem, we implicitly assumed that we were working on a flat plane. In this problem, we extend the triangulation problem so that the two robots may be on two different planes.

## Conclusion

I encourage Computer Science faculty teaching Robotics classes to consider incorporating a taste of this lower-level Robotics to their students. It can be done in a relatively short time and exposes the students to another important aspect of Robotics.

## References

[1] Kay, Jennifer, Introduction to Homogeneous Transformations and Robot Kinematics, Rowan University Computer Science Department, 2005. Available online at <http://elvis.rowan.edu/~kay/papers/kinematics.pdf> Slides to accompany this material are available online at <http://elvis.rowan.edu/~kay/robotics/slides/>

[2] Kelly, Alonzo, Essential Kinematics for Autonomous Vehicles, CMU-RI-TR-94-14, The Robotics Institute, Carnegie Mellon University, 1994. Available online at [http://www.frc.ri.cmu.edu/~alonzo/pubs/reports/pdf\\_files/kinematics.pdf](http://www.frc.ri.cmu.edu/~alonzo/pubs/reports/pdf_files/kinematics.pdf)