Abstracts of Talks at the Conference on Modular Forms and Related Topics in Honor of Marvin Knopp's 73rd Birthday

Temple University, Center City Campus
Philadelphia, PA
January 9 and 10, 2006

Monday January 9

9:00 - 9:40 **Bruce Berndt,** University of Illinois Urbana-Champaign
The Five Strangest, Most Fascinating, Most Interesting Results From Ramanujan's Lost Notebook (In the Speaker's Most Humble Opinion)
We discuss five results, or series of results, from Ramanujan's lost notebook that no one, it seems, but Ramanujan could have discovered. All of these results have further ramifications or generate further questions. The audience will be asked to complete ballots ordering their fascinations of these results from 1 to 5.

9:45 – 10:25 **Leon Ehrenpreis**, Temple University
Happy birthday, Marvin! Thanks for introducing me to the beauties of modular forms.
What did Marvin teach me about automorphic functions?
Marvin introduced me to the beautiful ideas of Rademacher, Zuckerman, etc. concerning explicit formulas for certain automorphic forms of nonpositive weight and Poicare series.I saw that these ideas could fit - to some extent- into my theory of fourier transform on varieties. The theory takes the following form:
Let G be and algebraic group defined over Q and let W be an arithmetic subgroup. Suppose we are given a representation B of G on R^n and that G preserves a nondegenerate quadratic form Q. Then Q allows us to define a G invariant Fourier transform. Suppose we have algebraic varieties V in x space and V* in Fourier transform space y. Let O be a discrete orbit of W on V which we identify with the sum of delta functions on O. Then we can take the Fourier transform of O. We can decompose O under various subgroups of G and the commutater of G. The interplay of such decompositions with corresponding decompositions in the Fourier transform space can be regarded as a generalization of the ideas of Marvin..

10:45 – 11:25 **Henryk Iwaniec**, Rutgers University
Extra Large Sieve for Hecke Eigenvalues
The large sieve is about asymptotic orthogonality of harmonics in an ambient space. The eigenvalues of Hecke operators are the harmonics which are powerful tools in analytic number theory. I will discuss the differences in various subfamilies, and how the results are applied.

10:45 – 11:25 **David James** (Joint work with John Clifford), University of Michigan-Dearborn
Composition Operators and Automorphic Functions,
Let φ denote an analytic mapping of a disc D into itself. One defines the composition operator C_φ acting on the space of functions analytic in D by

$C_\varphi(f) = f \circ \varphi$. A weighted composition operator with weight $w(z)$ satisfies $C_\varphi(f)(z) = w(z)[f \circ \varphi(z)]$. Inducing functions $\varphi(z) = \frac{az+b}{cz+d}$ which are linear fractional provide a rich approach to the study of composition operators. During the past thirty years, active interest in composition operators has developed to the extent that this area now has its own Mathematical Subject Classification (47B33), and involves a variety of issues such as compactness, eigenfunction properties, spectra, and semigroup properties, relating in particular the function theoretic properties of the inducing function φ to operator theoretic properties of C_φ for various classical Banach spaces of analytic functions on D.

This paper deals with the eigen properties of weighted composition operators. The eigenfunction equation for a composition operator C_φ is

$$C_\varphi(f) = \lambda f$$

Substituting $v(z)$ for $\lambda / w(z)$, we have

$$f \circ \varphi(z) = v(z) f(z)$$

valid for all z in a disc D. In almost all cases, any $f(z)$ satisfying the functional equation above can have its domain extended to a disc, a halfplane, plane or punctured plane $\hat{D}$ where φ maps $\hat{D}$ onto $\hat{D}$ (rather than into). If a fixed point of φ is interior to $\hat{D}$, this result can be used to classify the f in each of the following cases: 1) $v(z) = \lambda$ independent of z, 2) linear $v(z) = \lambda(z - z_1)$, 3) linear fractional $v(z) = \lambda \frac{(z - z_1)}{(z - z_2)}$, 4) general rational $v(z)$. When $\hat{D}$ is a disc or halfplane the f in the first case is an automorphic function. In all four cases the set of possible functions f is classified.

2:00 – 2: 40 **Geoffrey Mason,** University of California, Santa Cruz
2-dimensional Vector-valued Modular Forms
We discuss vector-valued modular forms for the modular group Γ and show how the holomorphic vector-valued modular forms corresponding to a 2-dimensional representation of Γ can be classified. Time permitting, we also discuss functional equations for the associated vector-valued *L*-series

2:45- 3:25 **Winfried Kohnen**, University of Heidelberg
Sign Changes of Fourier Coefficients and Eigenvalues of Cusp Forms
We will report on recent joint work with J. Sengupta, in which we give a uniform bound in terms of the weight and level of the first sign change of the Hecke eigenvalues of a normalized newform.

4:00 – 4:40 **Sol Friedberg**, Boston College
Multiple Dirichlet Series and Gauss Sums
In this lecture I will discuss a family of multiple Dirichlet series that are built out of sums of n-th order Gauss sums and a given root system of rank r. The combinatorics of the root system plays a key role in the definition. If n is sufficiently large, we show that these multiple Dirichlet series, originally defined in a product of r right half planes, have meromorphic continuation to C^r and satisfy a group of functional equations isomorphic to the Weyl group of the root

system. This work is all joint with Brubaker and Bump, and parts are also joint with Chinta and Hoffstein.

4:45 – 5:15 **Wladimir Pribitkin,** College of Staten Island
Five "Easy" Pieces Dedicated to My Teacher and Friend Marvin Knopp
Euler-Siegel, Dirichlet-Landau, Riemann-Selberg, Poincare-Petersson, Lehmer-Knopp

Tuesday January 10

9:00 – 9:40 **Dorian Goldfeld,** Columbia University
The Gelbart-Jacquet Lift
One of the main consequences of the Gelbart-Jacquet lift is that the Rankin-Selberg convolution of a Maass form on GL(2) with itself (divided by the Riemann zeta function) is the L-function of a Maass form on GL(3). We shall discuss a new elementary proof (joint work with Meera Thillainatesan) of this important result. As a corollary we obtain (joint work with Xiaoqing Li) a new simple proof of the Voronoi formula for GL(3) which is first obtained by Miller-Schmid (to appear in the Annals of Math).

9:45 – 10:25 **Thomas Schmidt,** Oregon State University
Low Height Geodesics on a Low-index Cover of the Modular Surface
In the 1950s, Harvey Cohn noticed the striking similarity between the Markoff equation $x^2 + y^2 + z^2 = 3xyz$ and Fricke's equation for parametrizing the space of hyperbolic once-punctured tori: $x^2 + y^2 + z^2 = xyz$. The connection allows the enumeration of the simple closed geodesics on certain low-index covers of the modular surface. In particular, the depth of penetration of a simple closed geodesic into the cusp is $\sqrt{9-4/z^2}$ with (x, y, z) a solution to Markoff's equation. All other geodesics penetrate further into the cusp, with those penetrating no further than region between 3 and $\sqrt{13}$ remaining a bit mysterious. In this report on joint work with M. Sheingorn, we describe certain general features of these low geodesics.

10:45- 11:25 **M. Ram Murty**, Queen's University (Canada)
The Lang-Trotter Conjecture
Let E be an elliptic curve and N_p, the number of points of E(mod p). Set a_p = p+1-N_p. In the 1970's, Lang and Trotter formulated precise conjectures concerning how often a_p takes on a fixed value. We will discuss modular versions of this conjecture and obtain various results using transcendental number theory. This is joint work with V. Kumar Murty.

11:30 – 12:00 **Abdul Hassen** (joint work with Hieu D. Nguyen), Rowan University
Hypergeometric Zeta Functions
In this talk, we investigate a new family of special functions referred to as hypergeometric zeta functions. Derived from the integral representation of the classical Riemann zeta function, hypergeometric zeta functions exhibit many

properties analogous to their classical counterpart, including the intimate connection to generalized Bernoulli numbers. We will demonstrate a functional inequality satisfied by the second-order hypergeometric zeta functions and discuss a zero free regions.

130 – 2:00 **Helen Grundman,** Bryn Mawr College
Arithmetic Genus and Rationality of Hilbert Modular Varieties
It is well-known that only finitely many Hilbert modular varieties are rational. We will discuss methods of narrowing the list of possibly rational varieties using both estimations and direct computations of the arithmetic genus.

2:05 – 3:35 **Sinai Robins,** Temple University
Integer Linear Programming Via Analytic Number Theoretic Functions Called Differentiable Dedekind Sums
In this talk, I will survey some history of Dedekind sums, and show the current development of a theory of higher-dimensional differentiable Dedekind sums, joint work with Helaman Fersugon. We use this theory to locate integer points in rational polytopes, one of the most fundamental problems in integer linear programming. Some examples in low dimensions will be given. Moreover, no background in Dedekind sums is assumed at all, to maximize accesibility for the casual observer.

3:00: 3:40 **Nikolaos Diamantis**, University of Nottingham (United Kingdom)
Eichler Cohomology and Second-Order Modular Forms
I will discuss recent joint work with C. O'Sullivan where we establish a first cohomological interpretation of the space of holomorphic second-order modular forms and prove precise formulas for its dimension.

3:45 – 4:15 **YoungJu Choie,** Pohang Institute of Technology (Korea)
Derivative of Modular Forms of Negative Weight
Let $f(\tau)$ be in the space of meromorphic modular forms of weight $-r$ with a positive integer r. Then it is known that the (r+1)th derivative $D^{r+1}f\left(=\frac{d^{r+1}f}{d\tau^{r+1}}\right)$ of f is again a modular form of weight r+2 and this is called "Bol's Identity"In this talk we discuss the Bol's identity for higher genus Siegel modular forms. We reveal some underlying representation theory behind Bol's identity and will formulate the purely representation – theoretic formula. This is done with D. Bump.

4:20 – 5:00 **George Andrews,** Penn State University
A Fine Dream
In his book, Basic Hypergeometric Series and Applications, Nathan Fine was able to develop many beautiful properties of q-series, partitions and mock theta functions from a careful study of first-order, non-homogeneous, linear q-difference equations. Fine had remarked during the preparation of his book that he was disappointed in not being able to encompass a proof of the Rogers-Ramanujan identities within this study. The object here is to show that a natural extension of Fine's methods leads directly to a new finite version of the Rogers-Ramanujan identities and, consequently, to a new proof of these celebrated formulas.