

## LINEAR ALGEBRA , WORKSHEET I

1. Consider the following system of linear equations.

$$2x + 2y - 2z - 2w = 4$$

$$3x + 5y + z + 2w = 0$$

$$-4x - 7y - 2z - w = 3$$

- a) Find the augmented matrix for the system of equations.  
b) Find the reduced row echelon form of the augmented matrix. c) What is the solution of the system of equations?

2. Consider a system of linear equations

$$x + z = 4$$

$$2x + y + 3z = 5$$

$$-3x - 3y + (a^2 - 5a)z = a - 5$$

For what value(s) of  $a$  does the system have (a) a unique solution, (b) infinitely many solutions, and (c) no solutions.

3. If  $A, B$  and  $C$  be matrices of size  $5 \times 3$ ,  $3 \times 9$  and  $5 \times 6$ , respectively, then what is the size of the matrix  $C^T(AB)$ ?

4. Suppose  $(2A)^{-1} = \begin{bmatrix} -3 & 5 \\ 1 & -2 \end{bmatrix}$ . Then find matrix  $A$ .

5. If  $k$  is a real number, then find the inverse of the matrix  $\begin{bmatrix} 1 & 0 & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ .

7. If  $A$  is an invertible  $n \times n$  matrix, then which of the following is true?

- I.  $\det(A) \neq 0$   
II. The system  $AX = 0$  has a unique solution.  
III. The reduced row-echelon form of  $A$  is  $I_n$ .

A) All but I   B) All but II   C) All but III   D) All   E) None

8. What is the set of value(s) of  $p$  for which the matrix  $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & p \\ -1 & 0 & 5 \end{bmatrix}$  is singular (or not invertible)?

9. If  $A$  and  $B$  are  $n \times n$  matrices that are invertible, prove that their product  $AB$  is also invertible.