

LINEAR ALGEBRA, WORKSHEET 2

1. Let $\mathbf{u} = (1, 7)$ and $\mathbf{v} = (3, -2)$. Compute each of the following.

a) $\mathbf{u} + \mathbf{v}$ b) $\|\mathbf{u} - \mathbf{v}\|$

2. What is the cosine of the angle between the vectors $\mathbf{u} = (1, -2, 3)$ and $\mathbf{v} = (-1, 1, 2)$?

3. a) Find the area of the triangle with vertices $P = (3, -5, 1)$, $Q = (2, 3, 1)$ and $R = (2, 2, 3)$.

b) What is the equation of the plane containing the points $P = (3, -5, 1)$, $Q = (2, 3, 1)$ and $R = (2, 2, 3)$?

4. Find the orthogonal projection of the vector $\mathbf{u} = (5, 6, -8)$ on the vector $\mathbf{a} = (0, -3, 4)$.

5. What is the distance between the line $3y = 4x + 5$ and the point $P = (1, 2)$?

6. Find a parametric equation of the line passing through the points $P = (3, 5, -1)$ and $Q = (2, 3, 4)$.

7. Find a vector that is orthogonal to both vectors \mathbf{u} and \mathbf{v} where $\mathbf{u} = (1, -1, 0)$ and $\mathbf{v} = (1, 0, -1)$.

8. What is the volume of the parallelepiped determined by the vectors $\mathbf{u} = (1, -1, 0)$, $\mathbf{v} = (1, 0, -1)$ and $\mathbf{w} = (3, 1, -2)$?

9. Find an equation of the plane through the point $P = (2, 7, -1)$ that is perpendicular to the line $x = 2 - 3t$, $y = 1 + t$ and $z = 2t$.

10. Is the vector $w = (1, -4, 30)$ in $\text{span}\{v_1, v_2\}$ where $v_1 = (1, 2, 8)$ and $v_2 = (3, 0, 1)$?

11. Show that the set of all points (x, y, z) such that $x + y - z = 0$ is a subspace of R^3 .

12. Consider the set of all vectors in R^3 of the form (a, b, c) where $a + b + c = 0$. Prove that this set is a subspace of R^3 .

13. Let $\mathbf{u} = (4, 2, 1, 3)$, $\mathbf{v} = (0, 3, 2, 0)$, $\mathbf{w} = (3, 2, 1, 1)$. Find

a) $\|\mathbf{u} - \mathbf{v}\|$ b) $\|\mathbf{u}\| - \|\mathbf{v}\|$ c) $\mathbf{u} \cdot \mathbf{v}$ d) What is the cosine of the angle between the vectors \mathbf{u} and \mathbf{v} ?

14. Let T be a function from R^3 to R^3 be given by $T(x, y, z) = (3x + y - z, x + z, y - 5z)$. Show that T is a linear transformation and find the standard matrix of T .