BOOK I.

DEFINITIONS.

1. A **point** is that which has no part.

2. A **line** is breadthless length.

3. The extremities of a line are points.

4. A **straight line** is a line which lies evenly with the points on itself.

5. A **surface** is that which has length and breadth only.

6. The extremities of a surface are lines.

7. A **plane surface** is a surface which lies evenly with the straight lines on itself.

8. A **plane angle** is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.

9. And when the lines containing the angle are straight, the angle is called **rectilineal.**

10. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is **right,** and the straight line standing on the other is called a **perpendicular** to that on which it stands.

11. An **obtuse angle** is an angle greater than a right angle.

12. An **acute angle** is an angle less than a right angle.

13. A **boundary** is that which is an extremity of anything.

14. A **figure** is that which is contained by any boundary or boundaries.

15. A **circle** is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another;

16. And the point is called the **centre** of the circle.

17. A **diameter** of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.

18. A **semicircle** is the figure contained by the diameter and the circumference cut off by it. And the centre of the semicircle is the same as that of the circle.

19. **Rectilineal figures** are those which are contained by straight lines, **trilateral** figures being those contained by three, **quadrilateral** those contained by four, and **multilateral** those contained by more than four straight lines.

20. Of trilateral figures, an **equilateral triangle** is that which has its three sides equal, an **isosceles triangle** that which has two of its sides alone equal, and a **scalene triangle** that which has its three sides unequal.

21. Further, of trilateral figures, a **right-angled triangle** is that which has a right angle, an **obtuse-angled triangle** that which has an obtuse angle, and an **acute-angled triangle** that which has its three angles acute.

22. Of quadrilateral figures, a **square** is that which is both equilateral and right-angled; an **oblong** that which is right-angled but not equilateral; a **rhombus** that which is equilateral but not right-angled; and a **rhomboid** that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called **trapezia**.

23. **Parallel** straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

POSTULATES.

Let the following be postulated:

1. To draw a straight line from any point to any point.

2. To produce a finite straight line continuously in a straight line.

3. To describe a circle with any centre and distance.

4. That all right angles are equal to one another.

5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

COMMON NOTIONS.

1. Things which are equal to the same thing are also equal to one another.

2. If equals be added to equals, the wholes are equal.

3. If equals be subtracted from equals, the remainders are equal.

[7] 4. Things which coincide with one another are equal to one another.

[8] 5. The whole is greater than the part.

DEFINITION 1.

Σημεῖόν ἐστιν, οὗ μέρος οὐθέν.
A point *is that which has no part.*

An exactly parallel use of *μέρος* (*ἐστί*) in the singular is found in Aristotle, *Metaph.* 1035 b 32 *μέρος μὲν οὖν ἐστὶ καὶ τοῦ εἴδους*, literally "There is a *part* even of the form"; Bonitz translates as if the plural were used, "Theile giebt es," and the meaning is simply "even the form is *divisible* (into parts)." Accordingly it would be quite justifiable to translate in this case "A point is that which is *indivisible into parts*."

Martianus Capella (5th c. A.D.) alone or almost alone translated differently, "Punctum est cuius pars *nihil* est," "a point is that a part of which is *nothing*." Notwithstanding that Max Simon (*Euclid und die sechs planimetrischen Bücher*, 1901) has adopted this translation (on grounds which I shall presently mention), I cannot think that it gives any sense. If a part of a point is *nothing*, Euclid might as well have said that a point is *itself* "nothing," which of course he does not do.

Pre-Euclidean definitions.

It would appear that this was not the definition given in earlier text-books; for Aristotle (*Topics* VI. 4, 141 b 20), in speaking of "*the* definitions" of point, line, and surface, says that they *all* define the prior by means of the posterior, a point as an extremity of a line, a line of a surface, and a surface of a solid.

The first definition of a point of which we hear is that given by the Pythagoreans (cf. Proclus, p. 95, 21), who defined it as a "monad having position" or "with position added" (*μονὰς προσλαβοῦσα θέσιν*). It is frequently used by Aristotle, either in this exact form (cf. *De anima* I. 4, 409 a 6) or its equivalent: e.g. in *Metaph.* 1016 b 24 he says that that which is indivisible every way in respect of magnitude and *quâ* magnitude but has not position is a *monad*, while that which is similarly indivisible and has position is a *point*.

Plato appears to have objected to this definition. Aristotle says (*Metaph.*

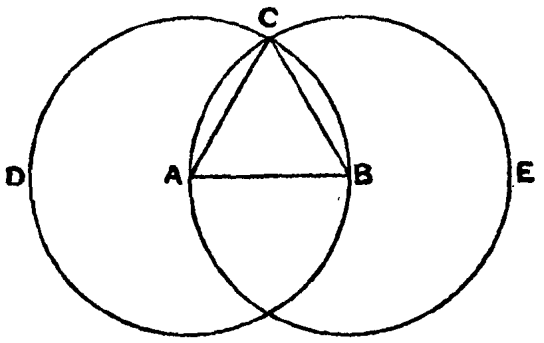
BOOK I. PROPOSITIONS.

PROPOSITION 1.

On a given finite straight line to construct an equilateral triangle.

Let AB be the given finite straight line.

Thus it is required to construct an equilateral triangle on the straight line AB.



With centre A and distance AB let the circle BCD be described; [Post. 3]

again, with centre B and distance BA let the circle ACE be described; [Post. 3]

and from the point C, in which the circles cut one another, to the points A, B let the straight lines CA, CB be joined. [Post. 1]

Now, since the point A is the centre of the circle CDB,

AC is equal to AB. [Def. 15]

Again, since the point B is the centre of the circle CAE,

BC is equal to BA. [Def. 15]

But CA was also proved equal to AB;

therefore each of the straight lines CA, CB is equal to AB.

And things which are equal to the same thing are also equal to one another; [*C. N.* 1]

therefore CA is also equal to CB.

Therefore the three straight lines CA, AB, BC are equal to one another.

Therefore the triangle ABC is equilateral; and it has been constructed on the given finite straight line AB.

(Being) what it was required to do.

1. **On a given finite straight line.** The Greek usage differs from ours in that the definite article is employed in such a phrase as this where we have the indefinite. *ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης*, "on *the* given finite straight line," i.e. the finite straight line which we choose to take.

3. **Let AB be the given finite straight line.** To be strictly literal we should have to translate in the reverse order "let the given finite straight line be the (straight line) AB"; but this order is inconvenient in other cases where there is more than one datum, e.g. in the *setting-out* of I. 2, "let the given point be A, and the given straight line BC," the awkwardness arising from the omission of the verb in the second clause. Hence I have, for clearness' sake, adopted the other order throughout the book.

8. **let the circle BCD be described.** Two things are here to be noted, (1) the elegant and practically universal use of the perfect passive imperative in constructions, *γεγράφθω* meaning of course "let it *have been* described" or "suppose it described," (2) the impossibility of expressing shortly in a translation the force of the words in their original order. *κύκλος γεγράφθω ὁ ΒΓΔ* means literally "let a circle have been described, the (circle, namely, which I denote by) BCD." Similarly we have lower down "let straight lines, (namely) the (straight lines) CA, CB, be joined," *ἐπεζεύχθωσαν εὐθεῖαι αἱ ΓΑ, ΓΒ*. There seems to be no practicable alternative, in English, but to translate as I have done in the text.

13. **from the point C....** Euclid is careful to adhere to the phraseology of Postulate 1 except that he speaks of "joining" (*ἐπεζεύχθωσαν*) instead of "drawing" (*γράφειν*). He does not allow himself to use the shortened expression "let the straight line FC be joined" (without mention of the points F, C) until I. 5.

20. **each of the straight lines CA, CB,** *ἑκατέρα τῶν* **ΓΑ, ΓΒ** and 24. **the three straight lines CA, AB, BC,** *αἱ τρεῖς αἱ* **ΓΑ, ΑΒ, ΒΓ.** I have, here and in all similar expressions, inserted the words "straight lines" which are not in the Greek. The possession of the inflected definite article enables the Greek to omit the words, but this is not possible in English, and it would scarcely be English to write "each of CA, CB" or "the three CA, AB, BC."

It is a commonplace that Euclid has no right to assume, without premising some postulate, that the two circles *will* meet in a point C. To supply what is wanted we must invoke the Principle of Continuity (see note thereon above, p. 235). It is sufficient for the purpose of this proposition and of I. 22, where there is a similar tacit assumption, to use the form of postulate suggested by Killing. "*If a line* [in this case e.g. the circumference ACE] *belongs entirely to a figure* [in this case a plane] *which is divided into two parts* [namely the part enclosed within the circumference of the circle BCD and the part outside that circle], *and if the line has at least one point common with each part, it must also meet the boundary between the parts* [i.e. the circumference ACE must meet the circumference BCD]."

Zeno's remark that the problem is not solved unless it is taken for granted that two straight lines cannot have a common segment has already been mentioned (note on Post. 2, p. 196). Thus, if AC, BC meet at F before reaching C, and have the part FC common, the triangle obtained, namely FAB, will not be equilateral, but FA, FB will each be less than AB. But Post. 2 has already laid it down that two straight lines cannot have a common segment.

Proclus devotes considerable space to this part of Zeno's criticism, but satisfies himself with the bare mention of the other part, to the effect that it is also necessary to assume that two *circumferences* (with different centres) cannot have a common part. That is, for anything we know, there may be any number of points C common to the two circumferences ACE, BCD. It is not until III. 10 that it is proved that two circles cannot intersect in more