A TREATISE OF THE METHOD of FLUXIONS AND INFINITE SERIES,

With its Application to the Geometry of CURVE LINES.

By Sir ISAAC NEWTON, Kt.

Tranſlated from the *Latin* Original not yet publiſhed.

Deſigned by the AUTHOR for the Uſe of LEARNERS.

Hac via inſiſtendum eſt.

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The Mathematical

OF

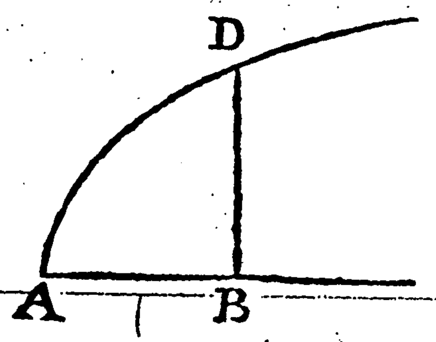
ANALYSIS

BY

Equations of an infinite Number of Terms.

1. *THE General Method, which I had devised some considerable Time ago, for measuring the Quantity of Curves, by Means of Series, infinite in the Number of Terms, is rather shortly explained, than accurately demonstrated in what follows.*

2. Let the Base AB of any Curve AD have BD for it's perpendicular Ordinate; and call AB $= x$, BD $= y$, and let a, b, c, &c. be given Quantities, and m and n whole Numbers. Then



The Quadrature of Simple Curves,

RULE I.

3. If $ax^{\frac{m}{n}} = y$; it shall be $\frac{an}{m+n}x^{\frac{m+n}{n}} =$ Area ABD.

The thing will be evident by an Example.

1. If $x^2 \left(= 1x^{\frac{2}{1}}\right) = y$, that is $a = 1 = n$, and $m = 2$; it shall be $\frac{1}{3}x^3 =$ ABD.

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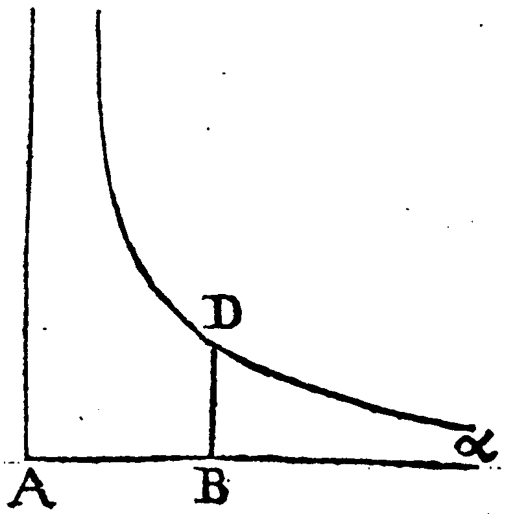
2. Suppose $4\sqrt{x}\ (=4x^{\frac{1}{2}}) = y$; it will be $\frac{8}{3}x^{\frac{3}{2}}\ (=\frac{8}{3}\sqrt{x^3}) =$ ABD.

3. If $\sqrt[3]{x^5}\ (=x^{\frac{5}{3}}) = y$; it will be $\frac{3}{8}x^{\frac{8}{3}}\ (=\frac{3}{8}\sqrt[3]{x^8}) =$ ABD.

4. If $\frac{1}{x^2} = x^{-2}) = y$, that is if $a = 1 = n$, and $m = -2$;



It will be $\frac{1}{-1}x^{\frac{-1}{1}} =) - x^{-1}\ (= \frac{-1}{x}) = \alpha$BD, infinitely extended towards α, which the Calculation places negative, because it lyes upon the other side of the Line BD.*

5. If $\frac{1}{\sqrt{x^3}}\ (x^{-\frac{3}{2}}) = y$; it will be $\left(\frac{2}{-1}x^{-\frac{1}{2}} =\right)$ $\frac{2}{-\sqrt{x}} =$ BDα.

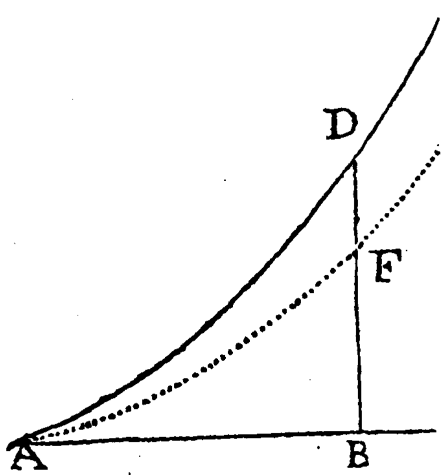
6. If $\frac{1}{x}\ (= x^{-1}) = y$; it will be $\frac{1}{0}x^{\frac{0}{1}} = \frac{1}{0}x^{0} = \frac{1}{0} \times 1 = \frac{1}{0} =$ an infinite Quantity; such as is the Area of the Hyperbola upon both Sides of the Line BD.

The Quadrature of Curves compounded of simple ones.

RULE II.

4. If the Value of y be made up of several such Terms, the Area likewise shall be made up of the Areas which result from every one of the Terms.

The first Examples.



5. If it be $x^2 + x^{\frac{3}{2}} = y$; it will be $\frac{1}{3}x^3 + \frac{2}{5}x^{\frac{5}{2}} =$ ABD.

For if it be always $x^2 =$ BF and $x^{\frac{3}{2}} =$ FD, you will have by the preceding Rule $\frac{1}{3}x^3 =$ Superficies AFB; described by the Line BF; and $\frac{2}{5}x^{\frac{5}{2}} =$ AFD described by DF; wherefore $\frac{1}{3}x^3 + \frac{2}{5}x^{\frac{5}{2}} =$ the whole Area ABD.

Thus if it be $x^2 - x^{\frac{3}{2}} = y$; it will be $\frac{1}{3}x^3 - \frac{2}{5}x^{\frac{5}{2}} =$ ABD. And if it be $3x - 2x^2 +$

12. Let $\frac{aa}{b+x}=y$; *Viz.* where the Curve is an Hyperbola.

Now that that Equation may be freed from it's Denominator, I make the Division thus.

$$b+x)\ aa+0\ \left(\frac{aa}{b}-\frac{aax}{b^2}+\frac{aax^2}{b^3}-\frac{aax^3}{b^4}\ \&c.\right.$$

$$aa+\frac{aax}{b}$$

$$0-\frac{aax}{b}+0$$

$$-\frac{aax}{b}-\frac{aax^2}{b^2}$$

$$0+\frac{aax^2}{b^2}+0$$

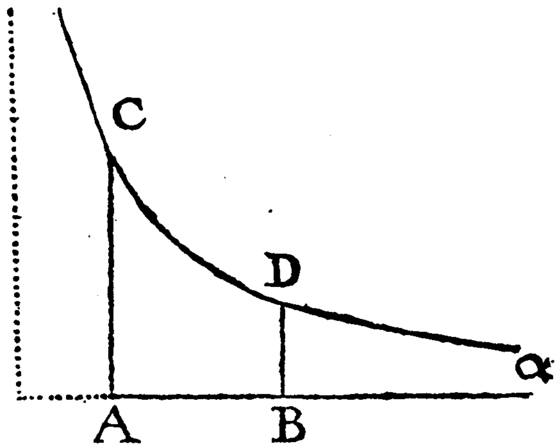
$$+\frac{aax^2}{b^2}+\frac{aax^3}{b^3}$$

$$0-\frac{aax^3}{b^3}+0$$

$$-\frac{aax^3}{b^3}-\frac{aax^4}{b^4}$$

$$0+\frac{aax^4}{b^4}$$

&c.



And thus in Place of this $y=\frac{aa}{b+x}$, a new Equation ariſes, *viz.* $y=\frac{a^2}{b}-\frac{a^2x}{b^2}+\frac{a^2x^2}{b^3}-\frac{a^2x^3}{b^4}$ &c. this Series being continued infinitely; and therefore (by the ſecond Rule)

The Area ſought ABDC will be equal to $\frac{a^2x}{b}-\frac{a^2x^2}{2b^2}+\frac{a^2x^3}{3b^3}-\frac{a^2x^4}{4b^4}$ &c. an infinite Series likewiſe, but yet ſuch, that a few of the initial Terms are exact enough for any Uſe, provided that b be equal to x repeated ſome few times.

13. After the ſame Manner if it be $\frac{1}{1+xx}=y$, by dividing there

14. Finally, if it be $\frac{2x^{\frac{1}{2}} - x^{\frac{3}{2}}}{1 + x^{\frac{1}{2}} - 3x} = y$; by dividing there arises $2x^{\frac{1}{2}} - 2x + 7x^{\frac{3}{2}} - 13x^2 + 34x^{\frac{5}{2}}$ &c. whence it will be
ABDC $= \frac{4}{3}x^{\frac{3}{2}} - x^2 + \frac{14}{5}x^{\frac{5}{2}} - \frac{13}{3}x^3$ &c.

Examples, where the Square Root must be extracted.

15. If it be $\sqrt{aa + xx} = y$, I extract the Root thus:

$$aa + xx \left(a + \frac{x^2}{2a} - \frac{x^4}{8a^3} + \frac{x^6}{16a^5} - \frac{5x^8}{128a^7} \text{ \&c.} \right.$$

$$aa$$

$$\overline{0 + x^2}$$

$$x^2 + \frac{x^4}{4a^2}$$

$$\overline{0 - \frac{x^4}{4a^2}}$$

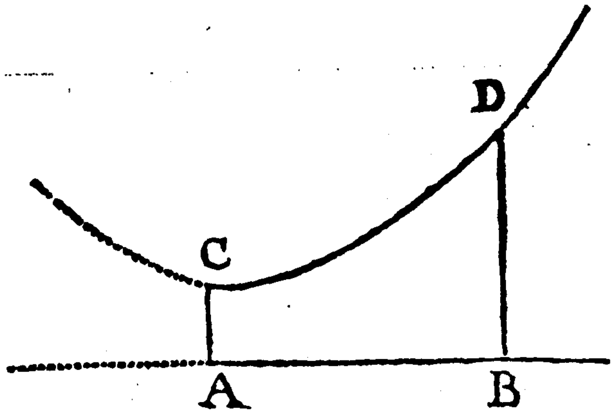
$$-\frac{x^4}{4a^2} - \frac{x^6}{8a^4} + \frac{x^8}{64a^6}$$

$$\overline{0 + \frac{x^6}{8a^4} - \frac{x^8}{64a^6}}$$

$$+\frac{x^6}{8a^4} + \frac{x^8}{16a^6} - \frac{x^{10}}{64a^8} + \frac{x^{12}}{256a^{10}}$$

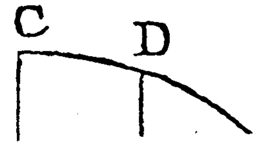
$$\overline{0 - \frac{5x^8}{64a^6} + \frac{x^{10}}{64a^8} - \frac{x^{12}}{256a^{10}}}$$

&c.



Whence for the Equation $\sqrt{aa + xx} = y$, a new one is produced, *viz.* $y = a + \frac{x^2}{2a} - \frac{x^4}{8a^3} + \frac{x^6}{16a^5} - \frac{5x^8}{128a^7}$ &c. And (by the second Rule) You will have the Area sought ABDC $= ax + \frac{x^3}{6a} - \frac{x^5}{40a^3} + \frac{x^7}{112a^5} - \frac{5x^9}{1152a^7}$ &c.

And this is the Quadrature of the Hyperbola.



16. After the same Manner if it be $\sqrt{aa - xx}$