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Ptolemy vs. Copernicus

Thomas J. Osler and Joseph Diaco
Mathematics Department
Rowan University
Glassboro, NJ 08028

Osler@rowan.edu

1. Introduction

Before recorded history, it was observed that the stars formed fixed patterns in the sky. These became our constellations. The ancients watched the sun, moon and five of the brightest stars (the planets) move through the fixed stars in a great band called the zodiac that circles the heavens. The familiar twelve signs from astrology are the twelve constellations that comprise this zodiac.

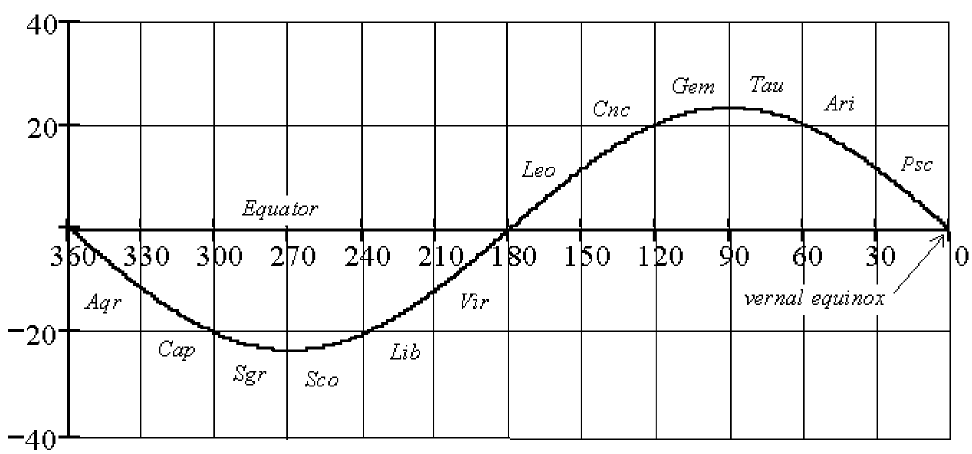

Figure 1: The Ecliptic and Constellations of the Zodiac as they appear on a celestial map

showing latitudes from –40 to 40 degrees.

The path that the sun takes through the zodiac is called the *ecliptic*. Imagine a map of the entire sky, much like a map of the world, only it shows the stars and constellations. Figure 1 is such a map (without the stars plotted). Here the ecliptic is the sinusoidal curve. (The signs of the zodiac are shown abbreviated.) The moon and planets move in paths that are very close to the ecliptic. To identify an object's position on the ecliptic, astronomers measure its distance in degrees from the vernal equinox, the point where the sun crosses the equator on the first day of spring. We will always denote this important angle by the variable θ.

About 150AD, Claudius Ptolemy published his *Almagest* [7,8] which was the bible of astronomy for 1500 years. In his cosmology, the Earth was at the centre of the universe. In section 2 we will use a simplified version of Ptolemy's cosmos to obtain equations for the position θ of a planet. In 1543 Nicolaus Copernicus, a Polish priest, published his *De Revolutionibus* [1,2] in which he placed the sun at the centre. In section 3 we use a simplified version of the Copernican view to derive equations for θ. Remarkably, we get the same equations that we obtained from Ptolemy’s geocentric universe! We discuss this in section 4. This analysis requires nothing more than familiarity with trigonometry; in particular, it does not use calculus.

2. The Simplified Ptolemaic View

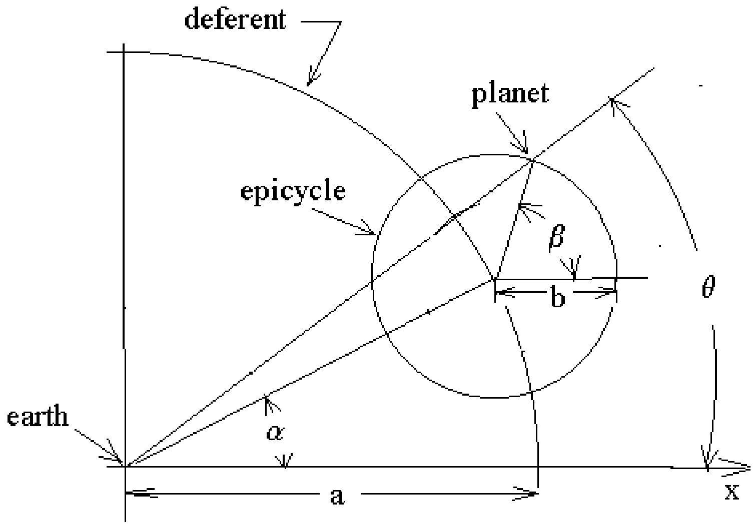

Figure 2: The simplified Ptolemaic system

Figure 2 shows the essential features of the Ptolemaic system. The ecliptic plane is shown in this figure. The planet is on the rim of a small circle of radius b called the epicycle. This circle rotates with constant angular velocity ω_E. The angle β is given by

(1) $$\beta = \omega_E t + \gamma_E ,$$

where t is time and γ_E is the value of β when $t = 0$. The centre of the epicycle in turn moves uniformly on the rim of a large circle of radius a called the *deferent*. The angle α is given by

(2) $$\alpha = \omega_P t + \gamma_P .$$

The deferent rotates with constant angular velocity ω_P and γ_P is the value of α when $t = 0$.

From Figure 2 it is easy to see that the x and y coordinates of the planet are found using

(3) $$x = a\cos(\omega_P t + \gamma_P) + b\cos(\omega_E t + \gamma_E) \;,$$

(4) $y = a\sin(\omega_P t + \gamma_P) + b\sin(\omega_E t + \gamma_E)$.

Since the earth is at the origin of coordinates, the position of the planet is seen as the angle

(5) $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

3. The Simplified Copernican View

In the Copernican view, the sun is at the centre of coordinates as shown in Figure 3. The earth is orbiting the sun on a circle of radius *b* while the planet is on a circle of radius *a*. The angles α and β are again given by relations (1) and (2).

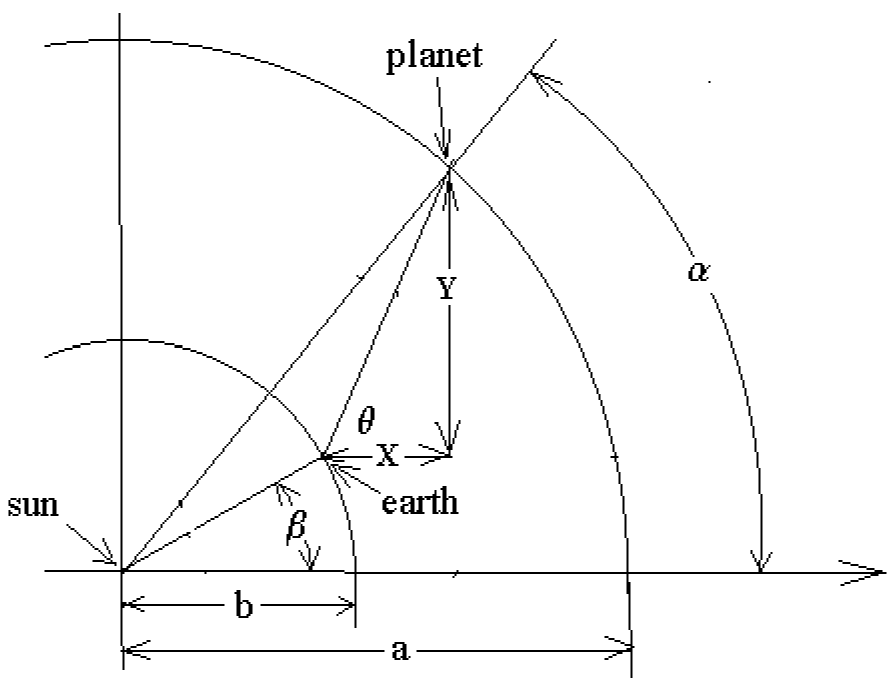

Figure 3: Simplified Copernican system

The values of *X* and *Y* are

(6) $X = a\cos(\omega_P t + \gamma_P) - b\cos(\omega_E t + \gamma_E)$, and

(7) $Y = a\sin(\omega_P t + \gamma_P) - b\sin(\omega_E t + \gamma_E)$.

The angle at which the planet is observed in the heavens from the observer on the earth is given by

(8) $$\theta = \tan^{-1}\left(\frac{Y}{X}\right).$$

4. Ptolemy vs. Copernicus, what's the fuss?

Historically, the Copernican hypothesis was very controversial. Early supporters of the theory faced the ridicule of those who believed that a moving earth would result in everything flying off into the air. Worse, there were religious objections to moving man from the central location of God's creation. Johannes Kepler was excommunicated from the Lutheran church, and Galileo was tried by the inquisition for supporting the Copernican view.

However, from a mathematical point of view, the two theories are equivalent. Compare the Ptolemaic equations (3), (4) and (5) with their equivalent Copernican formulas (6), (7) and (8). The only difference is the minus signs in (6) and (7). If we replace γ_E by $\gamma_E + \pi$ in (6) and (7), then the minus signs disappear, and the Ptolemaic and Copernican formulas for the position of a planet become identical. The addition of π simply changes the reference angle from the positive x direction to the negative x direction. Thus from a mathematical point of view, there is no difference in the Ptolemaic and the Copernican systems.

This means that the geocentric and the heliocentric descriptions of planetary motion are equally accurate (or inaccurate) for calculating the positions of the planets as observed from the earth. (We say inaccurate because Kepler showed that the ellipse, and not the circle, was needed to describe the motion with better precision.)

In Figures 2 and 3 we have described the situation for a *superior planet*, that is, a planet rotating outside the orbit of the earth. We could easily have done the same for an *inferior* planet. Look at the two circles in the Ptolemaic system (Figure 2), and compare them to the two circles in the Copernican system (Figure 3). Notice that the the deferent circle becomes the orbit of the planet, while the epicyclic circle becomes the orbit of the earth. One outstanding advantage of the Copernican system is that it enabled astronomers to determine the relative size of the solar system. That is, assuming the radius of the orbit of the earth to be one, they could easily determine the size of the orbits of the other planets. This measure of the size of the known solar system was impossible for Ptolemy. You can read the actual calculations of Copernicus in [2] for Mercury (p. 287), Venus (p. 277), Mars (p. 275), Jupiter (p. 266) and Saturn (p. 255).

5. **Final remarks**

The original systems of Ptolemy and Copernicus are more involved than the simplified versions presented above. For a good description of the original systems see the book by Crowe [3]. Gingrich has many interesting discussions of the work of Ptolemy and Copernicus in [4] and [5]. The remarkable book by Price [6] discusses attempts to build mechanical calculators to implement the theories of Ptolemy during the middle ages.

References

[1] Copernicus, Nicolaus, *On the Revolutions of the Heavenly Spheres,* (translated by C. G. Wallis), Vol. 16 of *Great Books of the Western World,* R. M. Huthins, editor, William Benton, Pub., Encyclopedia Britanica, Inc., Chicago, 1952, pp. 505-838.

[2] Copernicus, Nicolaus, *On the Revolutions of the Heavenly Spheres,* (translated by C. G. Wallis), Prometheus Books, New York, N. Y., 1995.

[3] Crowe, Michael J., *Theories of the World – From Antiquity to the Copernican Revolution*, (2nd Revised Edition), Dover Publications, Minneola, N.Y., 2001.

[4] Gingerich, Owen, *The Great Copernicus Chase and other adventures in astronomical history*, Sky Publishing and Cambridge U. P., Cambridge, MA., 1992.

[5] Gingerich, Owen, *The Eye of Heaven – Ptolemy, Copernicus, Kepler*, The American Institute of Physics, New York, N.Y., 1993.

[6] Price, Derek J., *The Equatorie of the Planetis*, Cambridge University Press, New York, N.Y., 1955.

[7] Ptolemy, Claudius, *The Almagest*, (translated by R. C. Taliaferro), Vol. 16 of *Great Books of the Western World,* R. M. Huthins, editor, William Benton, Pub., Encyclopedia Britanica, Inc., Chicago, 1952, pp. 5-465.

[8] Ptolemy, Claudius, *Ptolemy's Almagest*, (translated by G. J. Toomer), Princeton University Press, Princeton, N. J., 1998.