REAL ANALYSIS
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# SOLUTIONS TO HW ON COUNTABLE SETS

2/

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x \in \mathbb{N} \Rightarrow$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | --- |
| $y \in \mathbb{Z} \Rightarrow$ | 0 | 1 | -1 | 2 | -2 | 3 | -3 | --- |

$$y = \frac{x}{2} \quad \text{IF } x \text{ IS EVEN}$$

$$y = -\frac{x-1}{2} \quad \text{IF } x \text{ IS ODD}$$

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3/

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x \in \mathbb{N}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | --- |
| $y \in \mathbb{E}$ | 0 | 2 | -2 | 4 | -4 | 6 | -6 | --- |

$$y = x \quad \text{IF } x \text{ IS EVEN}$$

$$y = -(x-1) \quad \text{IF } x \text{ IS ODD}$$

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4/ LET A & B BE COUNTABLE SETS

$$A = \{a_1, a_2, a_3, ---\}$$

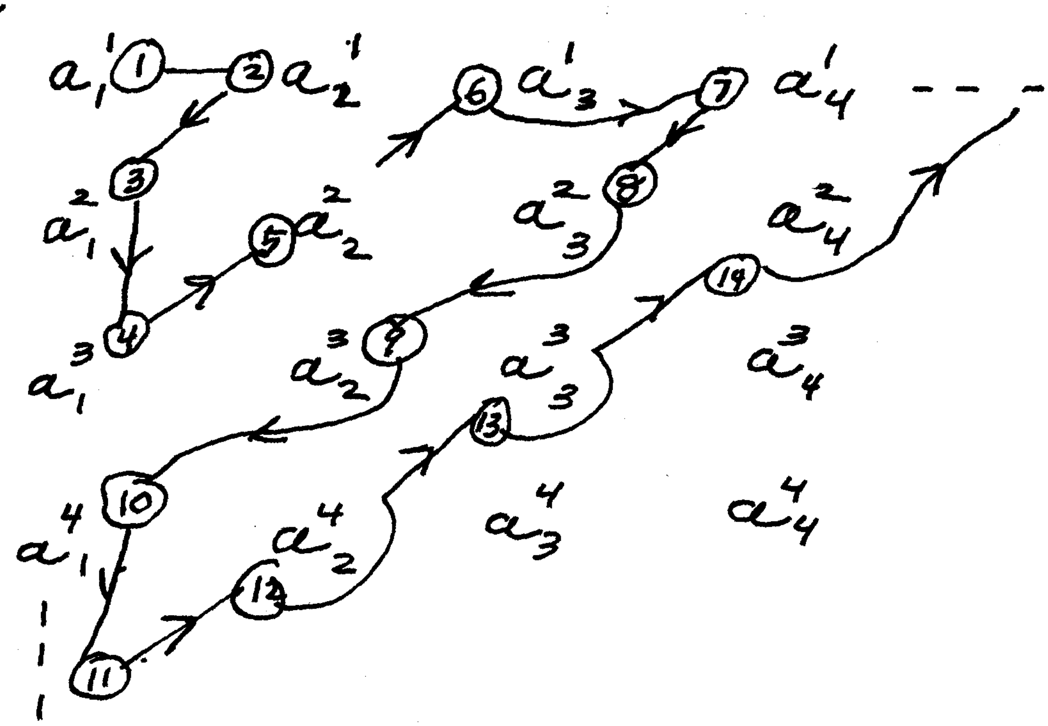
$$B = \{b_1, b_2, b_3, ---\}$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x \in \mathbb{N} \Rightarrow$ | 1 | 2 | 3 | 4 | 5 | 6 | --- |
| $y = A \cup B \Rightarrow$ | $a_1$ | $b_1$ | $a_2$ | $b_2$ | $a_3$ | $b_3$ | --- |

$$y = b_{\frac{x}{2}} \quad \text{IF } x \text{ IS EVEN}$$

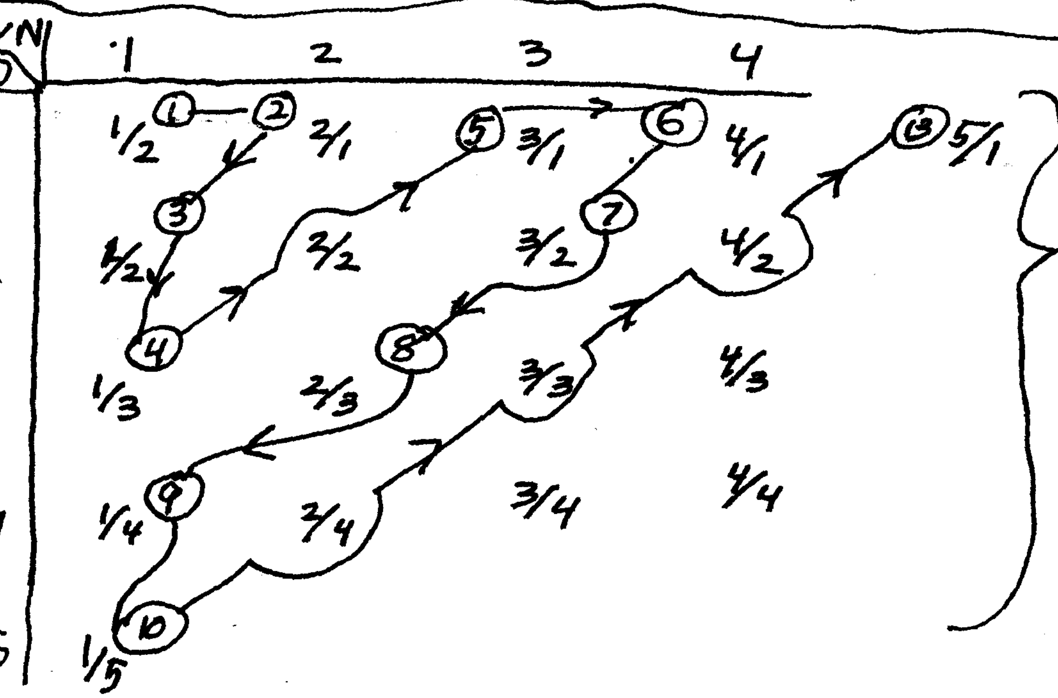
$$y = b_{\frac{x+1}{2}} \quad \text{IF } x \text{ IS ODD}$$

5/ CS 2




WE CAN MOVE THRU $\bigcup_1^\infty S_n$ IN THIS WAY ASSIGNING AN ELEMENT OF N TO EACH ELEMENT OF $\bigcup_1^\infty S_n$.

6/




ALL RATIONAL LISTED HERE $\frac{N}{D}$.

WE SKIP A RATIONAL IF IT WAS COUNTED PREVIOUSLY.

7/ EVERY X IS A DECIMAL OF THE FORM 0,7892--- STARTING WITH. 0.~~~~ , ASSUME BY CONTRADICTION THAT THE SET OF REALS IN $0<x<1$ IS COUNTABLE!

1 — 0,7289---- $= x_1$

2 — 0,2210---- $= x_2$

3 — 0,3431---- $= x_3$

4 — 0,1121---- $= x_4$

⋮ ⋮

DEFINE A NUMBER $y$ WHICH

(0) STARTS WITH 0.---

(1) DIFFERS FROM $x_1$ IN $1^{ST}$ DEC. DIGIT

(2) " " $x_2$ $2^{ND}$ "

(3) " " $x_3$ $3^{RD}$ "

(4) " " $x_4$ $4^{TH}$ "

⋮

$y = 0,6120---$

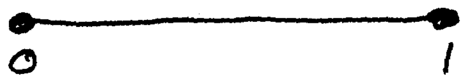
WE SEE THAT $y$ IS NOT IN OUR LIST, WHICH IS A CONTRADICTION,

8 F
9 F
10 F
12 T
13 T
14 T
15 F
17 F



16/ THE CANTOR TERNARY SET.

START WITH $[0,1]$




(STEP 0)

REMOVE THE OPEN MIDDLE THIRD


(STEP 1)

REMOVE THE OPEN MIDDLE THIRDS OF THESE TWO SEGMENTS


(STEP 2)

CONTINUE REMOVING OPEN MIDDLE THIRDS WITHOUT END. WHAT REMAINS IS CALLED THE CANTOR TERNARY SET.

CONSIDER WRITING THE NUMBERS AT EACH STEP IN BASE 3. (DIGITS 0, 1, 2)

EVERY NUMBER IN STEP 1 HAS ONLY A "0" OR A "2" IN THE FIRST DECIMAL PLACE. EVERY NUMBER IN STEP 2 HAS ONLY "0" OR "2" IN THE FIRST TWO DECIMAL PLACES. --- EVERY NUMBER IN STEP $n$ HAS ONLY "0" OR "2" IN THE FIRST $n$ DECIMAL PLACES.

THUS THE CANTOR SET CONSISTS OF ALL BASE THREE NOS. STARTING WITH 0.~~~~ AND USING <u>NO</u> ONES.