

Homework on Countable and Uncountable Sets

1. Memorize the following definition: A set is called *countable* or *denumerable* if it can be placed in one to one correspondence with the set of natural numbers, $\{1,2,3,4, \dots\}$
An equivalent definition says that a set is countable if its elements can be arranged in a sequence.
2. Show that the set of integers is countable.
3. Show that the set of even numbers is countable.
4. Show that the union of two countable sets is countable.
5. Let $S_1 = \{a_1^1, a_2^1, a_3^1, \dots\}$, $S_2 = \{a_1^2, a_2^2, a_3^2, \dots\}$, $S_3 = \{a_1^3, a_2^3, a_3^3, \dots\}$, ... be a countable collection of countable sets. Show that the union of these sets: $\bigcup_{n=1}^{\infty} S_n$.
6. Show that the collection of rational numbers is countable.
7. Show that the set of real numbers x satisfying $0 < x < 1$ is uncountable.
8. (T/F) The set of all real numbers is countable.
9. (T/F) The set of all points in the plane is countable.
10. (T/F) The set of all points in R^3 is countable.
11. Memorize the following definition: Two sets are said to be equivalent and to have the same cardinality if they can be put in one to one correspondence.
12. (T/F) The set of all points in R^n has the same cardinality as R .
13. (T/F) The set of all subsets of the infinite set S has cardinality greater than that of S .
14. (T/F) The set of all algebraic numbers is countable.
15. (T/F) The set of all transcendental numbers is countable.
16. Define the Cantor ternary set.
17. (T/F) The Cantor ternary set is countable.