AC Circuits: Driven Oscillations and Resonance

Introduction

In an AC (alternating current) circuit the current in the circuit alternates direction as a function of time. Since the current is time dependent, both capacitors and inductors play important roles in AC circuits. The response of an AC circuit will also depend on the driving frequency of the AC power supply.

Equipment

protoboard with AC supply	Capacitance decade box	Resistance decade box	Inductances
Alligator clips	oscilloscope	scope probes	

Theory

Some important AC circuit elements are:

a) Capacitors:

A capacitor stores charge +Q and -Q on either plate. Voltage is proportional to the charge on the plates. Potential energy is stored in the electric field between the plates. Because the voltage depends on charge rather than current, the voltage in the capacitor is out of phase with the current (voltage V_C lags current by $\pi/2$ when there is no resistance).

b) Inductors:

Changing current in an induced *emf* or voltage that is proportional to the rate of change of current. Potential energy is stored in the magnetic field created by the current. Because the voltage depends on the derivative of current, the voltage in the inductor is out of phase with the current (voltage V_L leads current by $\pi/2$ when there is no resistance).

c) Resistors

A resistor has voltage proportional to current (Ohm's Law: $V_R = IR$) Energy is dissipated in the resistor. The voltage on a resistor, V_R, is in phase with the current

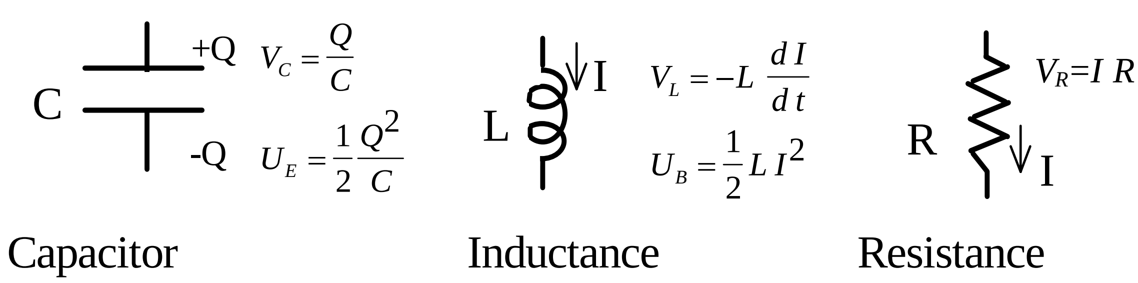

Figure 1: AC circuit elements. Voltage and energy in a capacitor depends on charge, which changes as current flows. Voltage and energy in an inductor depend on changes in the current. The resistor is a passive circuit element: voltage is directly proportional

The AC circuit: Driven oscillations and resonance.

An AC circuit driven by a sinusoidally varying power supply has a sinusoidally varying current and voltage. In the different circuit elements, current is not always "in phase" with voltage. That is, current may reach its maximum value at a different time than voltage reaches its maximum value.

Figure 2 shows how the voltage and current can vary in an AC circuit. The magnitude of the current and the phase difference between the voltage and current depend on circuit elements such as resistors, capacitors and inductors.

The relationship between peak voltage and peak current is characterized by the *impedance*, *Z*.

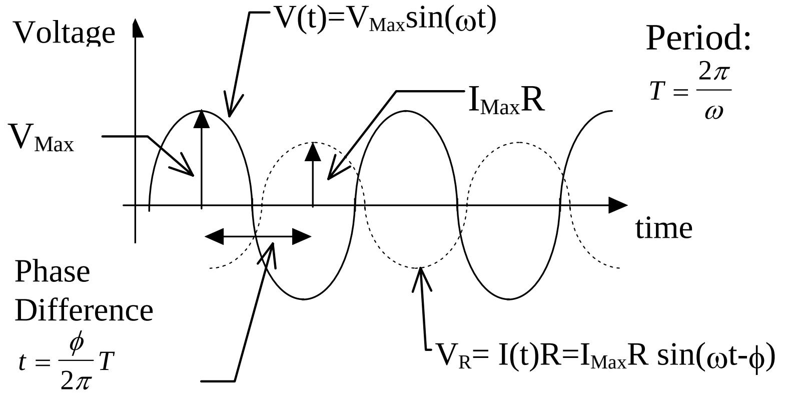

Figure 2: AC circuit. Solid line shows voltage from AC power supply, Dashed line shows voltage in resistor, which is in phase with the current. There is voltage and current peak at different times. The voltage leads the current here.

$$V_{Max} = I_{Max} \cdot Z$$

If the circuit is driven by a power supply with peak voltage V_{max} and angular driving frequency ω_d then

$$V(t) = V_{Max}\sin(\omega_d \cdot t) \qquad \text{and} \qquad I(t) = I_{Max}\sin(\omega_d \cdot t - \phi)$$

The smaller the impedance, the larger the current. The impedance, *Z*, and the phase depend not only on the resistance, capacitance and inductance in the circuit, but also on the driving frequency.

Resonance in the Series RLC circuit

A series RLC circuit is shown in Figure 3. For this circuit:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \text{and} \qquad \tan\phi = \frac{(X_L - X_C)}{R}$$

where

R = Resistance

X_L = Inductive Reactance = $\omega_d \cdot L$

X_C = Capacitive Reactance = $\dfrac{1}{\omega_d \cdot C}$

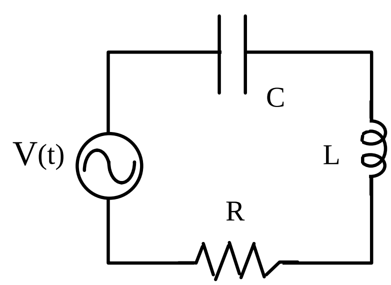

Figure 3: Series RLC circuit

The peak current in the circuit depends on the frequency of the driving voltage. For small frequencies the capacitor has a large reactance and for large frequencies the inductive reactance becomes large. The peak current is given by

$$I_{Max} = \frac{V_{Max}}{Z} = \frac{V_{Max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

so the largest value of peak current can be achieved when $X_L = X_C$ which is when $\omega_d \cdot L = 1/(\omega_d \cdot C)$. Notice that this occurs when the driving frequency ω_d is equal to the "natural" oscillating frequency of the series LC circuit, $\omega_r = 1/\sqrt{LC}$. This situation is known as *Resonance* and the frequency $\omega_r = 1/\sqrt{LC}$ is called the resonant frequency of the circuit. If the resistance is small, the peak voltage can become quite large. In fact, in the limit that the resistance is zero, the peak current would theoretically be infinite. In reality the power supply would limit the current.

Procedure

Setting up the oscilloscope

You will begin by setting up the oscilloscope to detect the sinusoidal voltage output by the protoboard. Connect Channel 1 of your scope to the sine wave output. All inputs should be set to AC and the trigger should be set to Channel 1. On the protoboard set the frequency to midrange on the 10 kHz scale and the amplitude to midrange. Set the time scale of your scope so that you can observe several periods of oscillation. Make sure all knobs on the scope are "clicked" into their calibrated positions. Adjust the position, intensity and trigger of your scope until you can see the voltage output from the protoboard. Ask for help if you are unsure. Use the position adjustments on your scope to verify which trace is CH 1 and which is CH 2. Vary the amplitude, frequency, and oscilloscope controls until you are sure you understand what you are seeing. Sketch what you see (including axis and division marks) and record the Volts/Division and Time/Division settings from the scope. Label V_{max} and the period, T. Calculate the frequency, f , and angular frequency, ω_d. Change the frequency by a factor of ten and re-adjust the scope. Measure the period and calculate the frequencies.

A capacitive circuit:

Set up the circuit shown in Figure 4 using the resistance and capacitance decade boxes. The small "m" on the capacitance decade box represents millifarads. Use the following settings:

R about 100 Ω	C about 0.0005 mF	*f* about 0.5 kHz

Connect Ch. 1 of the scope across the voltage source and Ch. 2 across the resistor. *Note that Ch. 1 and Ch. 2 actually have the same ground which must be connected to the same "ground" in the circuit.* Since the voltage on the resistor is $V_R = IR$, Ch. 2 will show the current in the circuit.

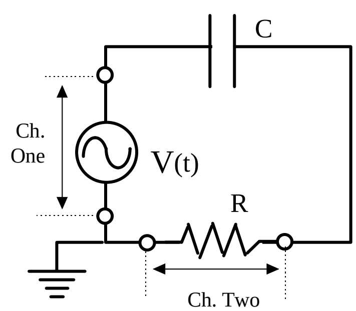

Figure 4: RC Circuit

Adjust your scope until you can see both Voltage (Ch. 1) and Current (Ch. 2). Adjust the Volts/div Time/div on the scope so that Voltage and Current both display well. Make a sketch of what you see, including axis and division markings. Record the Volt/div for both channels and the Time/Div. Measure the maximum voltages, period and calculate the maximum current and frequencies. On your sketch label amplitudes, V_{Max} and $V_R=I_{Max}R$, the period, T, and the time corresponding to the phase difference, $t = (\phi/2\pi)T$. (See Figure 2). From the amplitudes for V_{Max} and V_R determine the impedance, *Z.*

Does the voltage leads the current (peaks before the current) or lag the current (peaks after the current)? Calculate theoretical values for the impedance and phase and compare to experimental values.

Vary the a) frequency and b) the capacitance C (or inductance L for the next section) and observe the effects on the amplitude of the current and the phase. Discuss whether or not the behavior is what you would expect from the theoretical equations for impedance and phase.

An inductive circuit:

Set up the circuit shown in Figure 5 using the resistance decade box and L=100mH inductance. Use the following settings

R about 100 Ω	L about 100 mH	*f* about 50 kHz

Repeat the measurements and observations you made for the RC circuit.

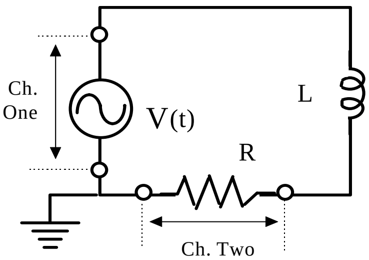

Figure 5: An LR circuit

Series RLC circuit and resonance

Set up the series circuit shown in Figure 6 so that it has a resonant frequency of about 10-100 kHz. Set the resistance to about 100 Ohms. Observe the current in the circuit as you vary the frequency. Find the resonant frequency for the circuit. Measure the amplitudes, period and phase. Calculate experimental values for the impedance and phase at the resonant frequency.

Repeat, substituting the 30mH impedance. Make a prediction for the new resonant frequency and then find it experimentally. At the resonant frequency measure the amplitudes, period and phase and compare the impedance and phase to the theoretical values.

Which way (earlier time or later time) does the peak current move when the driving frequency is less than the resonant frequency. When it is greater than the resonant frequency?

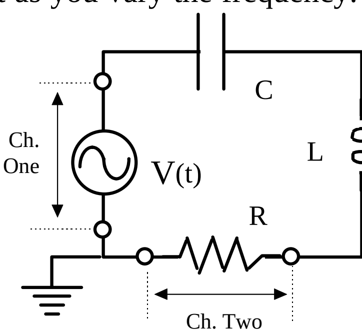

Figure 6: Series RLC circuit

Width of the resonance

Change the resistance to about 1000 Ohms. Vary the frequency around resonance and observe and differences in comparison to the circuit with 100 Ohms resistance. Notice both changes in the magnitude of the resonance and in its “width” (does it change a great deal over a short range of frequency (a “sharp” resonance) or does it change more slowly (a “broad” resonance).

Questions

Compare the capacitive circuit to the inductive circuit. For each, discuss how the impedance and phase changed with frequency. Discuss both your experimental observations (of amplitude and phase) and what the theory would predict.

For the RLC circuit, which way (earlier time or later time) does the peak current move when the driving frequency is less than the resonant frequency. When it is greater than the resonant frequency? Is this what you expect? Is the capacitor or the inductor more important at high and at low frequency.

Discuss how the height and width of the resonance depend on the resistance in the RLC circuit.

Capacitive (RC) circuit

Sketch:

Oscilliscope Settings:
Time/Div:

Ch. 1: Volts/Div

Ch. 2: Volts/Div

R= C=

Experimental (from oscilliscope trace) -- *include units*

T=	f =	$\omega =$
V_{Max}=		
V_R =	$I_{Max}=V_R/R$	$Z= V_{Max}/ I_{Max}=$
t (for phase difference)=		$\phi = 2\pi \cdot (t/T) =$

Theoretical Values (give formula)

Z=	% Diff.
$\phi =$	% Diff.

Does the voltage lag or lead the current? How does the current amplitude and phase change with frequency?

Inductive (RL) circuit

Sketch:

Oscilliscope Settings:
Time/Div:

Ch. 1: Volts/Div

Ch. 2: Volts/Div

R= L=

Experimental (from oscilliscope trace) -- *include units*

T=	f =	$\omega =$
$V_{Max}=$		
V_R =	$I_{Max}=V_R/R$	$Z= V_{Max}/ I_{Max}=$
t (for phase difference)=		$\phi = 2\pi \cdot (t/T) =$

Theoretical Values (give formula)

Z=	% Diff.
$\phi =$	% Diff.

Does the voltage lag or lead the current? How does the current amplitude and phase change with frequency?

Series RLC circuit

R= L= C=

Experimental (from oscilliscope trace) -- *include units*

T=	f =	ω_d=
V_{Max}=		
V_R =	$I_{Max}=V_R/R$	$Z= V_{Max}/ I_{Max}$=
t (for phase difference)=		$\phi = 2\pi \cdot (t/T) =$

Theoretical Values (give formula)

ω_r=	% Diff.
Z=	% Diff.
ϕ=	% Diff.

Series RLC circuit

R= L= C=

Experimental (from oscilliscope trace) -- *include units*

T=	f =	ω_d=
V_{Max}=		
V_R =	$I_{Max}=V_R/R$	$Z= V_{Max}/ I_{Max}$=
t (for phase difference)=		$\phi = 2\pi \cdot (t/T) =$

Theoretical Values (give formula)

ω_r=	% Diff.
Z=	% Diff.
ϕ=	% Diff.