1. Percent Difference

When comparing an experimental value to a theoretical value it is usually more useful to consider the percent difference rather than the absolute numerical difference. The percent difference is give by

$$\%\text{Difference} = \frac{Experimental - Theoretical}{Theoretical} \times 100\%$$

The percent difference will be positive if the experimental value is greater than theoretical and negative if it is less than the theoretical value. The theoretical value is assumed to be well-known, which is why it is in the denominator.

When comparing two experimental values we may not know which is more accurate. In this case we need to calculate the average

$$Average = (ExperimentalOne + ExperimentalTwo)/2$$

and then we can find the percent difference between the two values by dividing by the average:

$$\%\text{Difference} = \frac{ExperimentalOne - ExperimentalTwo}{Average} \times 100\%$$

This does not "favor" either experimental value. The sign in this case is not meaningful since we are not sure which experimental value might be more accurate.

2. Uncertainties and significant figures

All measured values should be reported as $x \pm \Delta x$, where x is the measured (or calculated) value and Δx is the uncertainty. (Other symbols used for uncertainty include σ_x and δx). The ratio $\Delta x/x$ is defined as the fractional uncertainty and $100\% \times \Delta x/x$ is the percent uncertainty.

In this course, uncertainties will be rounded to one significant figure. The last significant figure in the value x should be in the same decimal position as the uncertainty. For example, if you measure the length of a metal rod using a ruler which has mm markings, you may be able to measure the length to within 0.5 mm. The length would be reported as $37.0\text{mm} \pm 0.5\text{mm}$ or perhaps as $3..70\text{cm} \pm 0.05\text{cm}$. Notice that the last significant figure in the data is the same size as the uncertainty.

Random uncertainties

Random uncertainties are inherent in the process of measurement. They can be reduced by making careful and/or repeated measurements. Uncertainty can be estimated by looking at how “scattered” the data is about the mean (average). For large numbers of measurements a more sophisticated estimate called the “standard deviation” is calculated.

If a measurement is made only once, the random uncertainty must be estimated based on how precisely you were able to read the measurement off of the provided scale. Often, this uncertainty is quoted as being one-half of the smallest division on the measuring device, as in the example above.

The standard deviation

If the measurement is repeated several times, the random uncertainty can be calculated statistically. Consider a measurement of a quantity, x, made N times, providing the measurements $x_1, x_2, x_3, \cdots x_N$. From these measurements, we report

$$\bar{x} = \frac{1}{N}\cdot\left(x_1 + x_2 + \cdots + x_N\right) = \frac{1}{N}\sum_{i=1}^{N} x_i \quad \text{and} \quad \sigma_x = \frac{1}{\sqrt{(N-1)}}\times\sqrt{\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2}$$

where $\bar{x}$ is the mean (also called the average) and σ_x is an estimate of the uncertainty called the "standard deviation of the mean".

Systematic uncertainties

Systematic uncertainties are inherent in the instruments that we use to obtain data. Accurately calibrated instruments can reduce it. Systematic uncertainties are often provided by the manufacturer of the equipment. I will give you these uncertainties when necessary. The systematic and random uncertainties for a given measurement are summed, such that $\Delta x = \Delta x_{random} + \Delta x_{systematic}$.

"Propogation" of uncertainty

When several measured data values are used to calculate a final value, the uncertainties associated with the experimental measurements must be combined to find the final uncertainty

Addition and subtraction

When adding or subtracting two measured values, the uncertainties are added.
Example:

If $c = a + b$ or $c = a - b$ then:

Addition
$$\begin{cases} c = (a \pm \Delta a) + (b \pm \Delta b) \Rightarrow c = a + b \pm (\Delta a + \Delta b) \\ \quad \text{or} \quad c = a + b \quad \text{and} \quad \Delta \text{c} = (\Delta a + \Delta b) \end{cases}:$$

Subtraction
$$\begin{cases} c = (a \pm \Delta a) - (b \pm \Delta b) \Rightarrow c = a - b \pm (\Delta a + \Delta b) \\ \quad \text{or} \quad c = a - b \quad \text{and} \quad \Delta \text{c} = (\Delta a + \Delta b) \end{cases}$$

Multiplication and division

When multiplying or dividing two measured values, the *fractional* uncertainties are added.

If $c = a \times b$ or $c = a/b$ then:

Multiplication
$$\begin{cases} c = (a \pm \Delta a) \times (b \pm \Delta b) \Rightarrow c = a \times b \pm a \times b \times \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) \\ \quad \text{or} \quad c = a \times b \quad \text{and} \quad \frac{\Delta c}{c} = \frac{\Delta a}{a} + \frac{\Delta b}{b} \end{cases}$$

Division
$$\begin{cases} c = (a \pm \Delta a)/(b \pm \Delta b) \Rightarrow c = a/b \pm (a/b) \times \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) \\ \quad \text{or} \quad c = a/b \quad \text{and} \quad \frac{\Delta c}{c} = \frac{\Delta a}{a} + \frac{\Delta b}{b} \end{cases}$$

3. Understanding Resistor Codes.

Resistors are often color coded with their values. There are usually four bands of color. The first two bands represent two digits and the third band represents additional powers of ten. The fourth band is either silver or gold, indicating the value is good to 10% or to 5%. The numbers represented are:

Color	Black	Brown	Red	Orange	Yellow	Green	Blue	Violet	Gray	White
Value	0	1	2	3	4	5	6	7	8	9

A resistor with bands of Brown, Blue, Red and Gold would signify the three digits 162 and have a value of $16 \times 10^2 = 1600$ Ohms.

4. Reading a Vernier

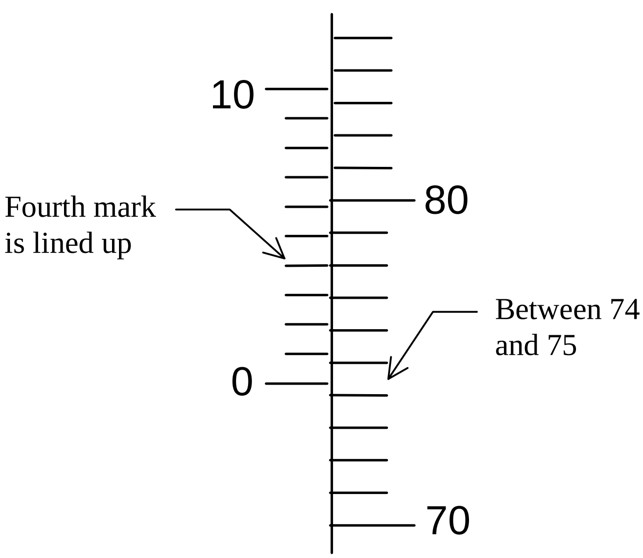

Figure 1: Vernier scales reading 74.4

Vernier scales consist of two scales side by side. One scale gives a rough scale reading and the second determines the decimal reading.

In the figure the zero of the left hand scale is lined up between 74 and 75 on the right hand scale. This tells us the reading is between 74 and 75.

To get the exact decimal reading look on the left hand scale to find where a marker lines up with one on the right hand scale -- this is the fourth line, so the reading should be 74.4.