

# Atwood's Machine: Newton's Second Law (approximately 2 hr.) (5/27/05)

## Introduction

A physical “law” is a statement of one of the fundamental theoretical principles that underlie our understanding of how the physical world works. Newton’s First and Second Laws describe the relationship between force, mass, and acceleration. The net (total) force on an object, which is the vector sum of all applied forces, is equal to the total mass times the acceleration of the object. In this lab you will use a simple system of pulleys, strings and weights to investigate this relationship.

## Equipment

- low friction pulley & clamp
- string
- 2 mass hangers
- 2-meter stick
- bench edge clamp
- level
- 2 sets of masses
- stopwatch
- 2 identical rubber bands
- ruler
- grid paper
- 0.9 m rod

## Before the Lab

Read the sections in your text describing Newton’s Laws. You should learn how to draw a force diagram (a.k.a. a “free-body” diagram) and determine the vector sum of all forces acting *on* an object.

## Theory

When two masses are suspended by a string over a pulley (Figure 1) each feels a downward force due to its weight ( $W=mg$ ) and an upward force due to the tension ( $T$ ) in the string (Figure 2). If these two forces are equal, then the net force on the mass is zero and, as Newton’s first law tells us, there will be *no acceleration* of the mass

If one mass is heavier than the other, then the masses may accelerate – one moving upward and the other down.

Newton’s second law tells us that in this case there must be a *net force* on each mass:

$$\vec{F}_{net} = m\vec{a}$$

If the string and pulley are considered to have negligible mass, then we can neglect the force needed to accelerate the string and cause the pulley to rotate. In this case the tension in the string is equal on both ends. The forces on the two masses are then:

$$F_1 = T - m_1g = m_1a \quad \text{and} \quad F_2 = T - m_2g = -m_2a$$

where we have used the fact that the acceleration of each mass is the same (since they are connected by the string) and assumed that mass 1 is accelerating upward and mass 2 is accelerating downward. Algebraically we can solve this to show that

$$m_2g - m_1g = (m_1 + m_2)a$$

In effect the tension pulling in opposite directions cancels, so the net force becomes

$$F_{net} = T - m_1g + m_2g - T = m_2g - m_1g \quad \text{while the total mass}$$

being accelerated is  $m_1 + m_2$ . Thus  $a = \frac{m_2 - m_1}{m_1 + m_2} g$  and if

$m_2$  is greater than  $m_1$  it will indeed accelerate downward. If  $m_1$  is greater, the acceleration is negative meaning that each mass accelerates in the direction opposite to what was originally assumed.

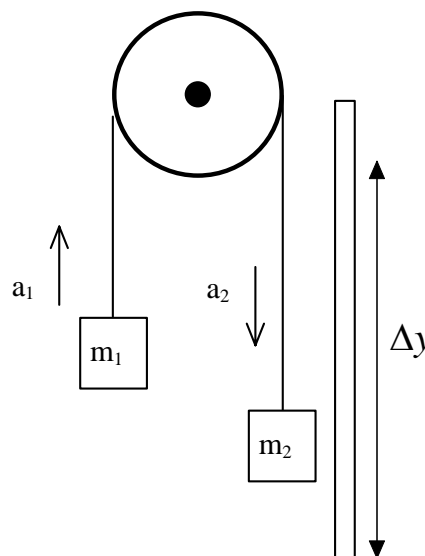


Figure 1: Mass and pulley system

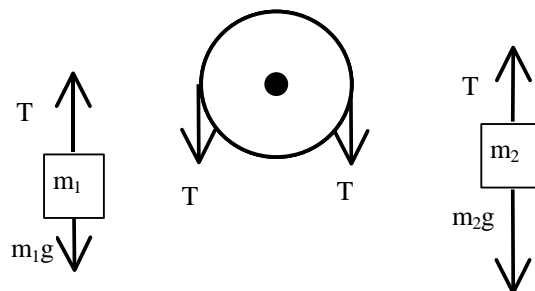


Figure 2: Force diagrams- draw each object separately

## **Friction and Rotational Inertia**

The real pulley and mass arrangement is not as simple as that described above. Some factors that will affect the acceleration are 1) friction, which has not been included in the net force, 2) the mass of the string, and 3) the *rotational inertia* of the pulley. If we add the force of friction,  $f$ , to the net force, it becomes  $F_{net} = m_2g - m_1g - f$ . The fact the pulley is not massless means that it does require a net “torque” (a “turning” force) to make it rotate – this is supplied by the tension in the string.

The *rotational inertia* of the pulley then adds an *equivalent mass* to the total mass being accelerated,

so that  $a = \frac{(m_2 - m_1)g - f}{m_1 + m_2 + m_{equiv}}$  where the equivalent mass for the pulley is approximately

$$m_{equiv} = \frac{1}{2} M_{pulley}. \text{ (Note: The mass of the pulley disc is approximately 100 g.)}$$

## **Determining the acceleration with a stopwatch and meter stick**

The acceleration may be determined by measuring the time,  $t$ , required for the mass to fall from rest a specific distance  $\Delta y$ . If  $a$  is constant then we know that  $\Delta y = v_0t + \frac{1}{2}at^2 = \frac{1}{2}at^2$ .

### ***Procedure***

You will be investigating the relationship between mass and acceleration by varying both  $m_1$  and  $m_2$  and observing  $a$ . We have shown from Newton’s Second Law that

$$a = \frac{(m_2 - m_1)g - f}{m_1 + m_2 + m_{equiv}}.$$

Thus it seems worthwhile to try to systematically investigate the relationship between mass, force and acceleration by 1) keeping the net force constant by holding  $(m_2 - m_1)g$  constant while varying  $(m_2 + m_1)$  and 2) keeping the total mass,  $(m_2 + m_1)$ , constant while varying the net force by changing  $(m_2 - m_1)g$

### **Vary the total mass (keep the net force constant)**

Set up the apparatus as shown in the figure. Before assembling it record the weights of the mass hangers, the masses you will use, the pulley and the string. Use a piece of string long enough that you can observe the mass falling at least 1 meter. Use pad to protect the floor from impacts.

Begin with the two masses  $m_1$  and  $m_2$  each equal to 50 grams (this may be the mass of the hangers alone). When the two masses are equal the net force is zero. In the absence of friction, if the masses are set in motion they should continue to move at constant speed (zero acceleration). Hang the masses on the pulley and if necessary tap one side to set it in motion. Do you observe any significant friction? Record your observations.

Add 10 grams to  $m_2$  but leave  $m_1$  fixed. Start with  $m_2$  near the pulley and measure the distance from the bottom of  $m_2$  to the floor. This is  $\Delta y$ . Then release  $m_2$  and record the time,  $t$ , which it takes to hit the ground. Make a few trial runs for practice and then record three independent measurements of  $t$ .

Add 100 grams to each side. This keeps the weight difference constant while increasing the total mass. Re-measure  $\Delta y$  (in case the string stretches) and record three independent measurements of time  $t$ .

Repeat three more times by adding additional increments of 100 grams to each side for a total of four trials with the 10-gram mass difference. Always re-measure  $\Delta y$ .

Now repeat for a total of 4 more trials using a 20-gram mass difference. (Note: Time permitting.)

### **Vary the net force (keep the total mass constant)**

Begin with a total of 260 grams for *each* mass. For the ascending mass,  $m_1$  use a combination of (50 + 200 + 5 + 2 + 2 + 1) grams. For the descending mass,  $m_2$ , use (50 + 200 + 10) grams.

(Question: Should the mass of the mass hanger be included here? Why or why not?)

Transfer 2 grams from  $m_1$  to  $m_2$ . Record the mass of each (ex. 258 grams and 262 grams) then measure  $\Delta y$  and make at least three good trials to record times and average.

Repeat by transferring additional mass from  $m_1$  to  $m_2$  in 1 gram increments:

Transfer an *additional* 1 g from  $m_1$  to  $m_2$  (i.e. 257 g and 263 g).

Transfer an *additional* 2 g from  $m_1$  to  $m_2$  (i.e. 256 g and 264 g, etc.)

Transfer an *additional* 3 g from  $m_1$  to  $m_2$ .

Transfer an *additional* 4 g from  $m_1$  to  $m_2$ .

Repeat, starting with a total of 360 grams for *each mass*. (Note: Time permitting.)

### **Analysis**

- 1) For each trial, calculate the experimentally determined acceleration from the distance and time measurements. Show how the calculation is performed.
- 2) From the masses and mass differences calculate the theoretical acceleration. Include the effect of the equivalent mass of the pulley but neglect the effect of friction.
- 3) Calculate the percent difference between the theoretical and experimental values. (Percent difference is equal to  $((Exp.-Theo.)/Theo.) \times 100\%$  .)
- 4) For the data with constant total mass, the predicted relationship between acceleration and mass difference :  $a = \frac{(m_2 - m_1)g - f}{m_1 + m_2 + m_{equiv}}$  can be represented on a graph of acceleration ( $a_{Exp.}$ ) versus mass difference ( $m_1 - m_2$ ). A graph of y versus x is a straight line if it fits the formula " $y(x)=mx+b$ ", where " $m$ " is the slope and " $b$ " is the intercept. Rewrite the above equation for  $a$  in this form. What is the slope of the graph of  $a_{Exp}$  versus  $(m_2 - m_1)$ ? What is the intercept. Answer these questions in your report.
- 5) Using a computer graphing program (EXCEL), make graphs of ( $a_{Exp.}$ ) versus  $(m_2 - m_1)$  for each of your two data sets. Use a least squares fitting procedure (linear trend line) to find the slope and intercept of your graph. Determine the units of the slope and intercept.
- 6) From the slope and intercept, determine the gravitational acceleration,  $g$  and the force of friction,  $f$ . Make sure your units come out in  $m/sec^2$  for acceleration and in Newtons for force.

### **Questions**

Be sure that your report documents your calculations and explains how the value of  $g$  and the magnitude of the frictional force  $f$  can be determined from the graph.

How does the fact that the string has mass affect your experiment? Do you consider this an important effect?

Compare the magnitude of the force of friction to the other forces present in this experiment. Is friction a significant factor?

When the net force (due to mass difference) remains constant but the total mass increases, what happens to the acceleration?

When the total mass remains constant but the net force increases, what happens to the acceleration?

Explain specifically whether the answers to the previous two questions reflect Newton's Second Law.

**Vary the total mass (keep the net force constant): 10 gram mass difference**

Mass of pulley \_\_\_\_\_  $m_{\text{equiv}}$  for pulley \_\_\_\_\_  
 Mass of string \_\_\_\_\_ Mass of hangers \_\_\_\_\_

Data

Ascending mass,	$m_1$							
Descending mass,	$m_2$							
Distance	$\Delta y$							
Time, $t$	1							
	2							
	3							
Time	Avg.							

Calculations

Calculated acceleration, $a_{\text{exp}}$								
Total mass $m_1 + m_2 + m_{\text{equiv}}$								
Mass Difference $(m_2 - m_1)$								
Net Force $(m_2 - m_1)g$								
Theoretical Acceleration, $a_{\text{theo}}$ .								
% Diff between exp. and theo.								

Calculations (show and explain work)

**Vary the total mass (keep the net force constant): 20 gram mass difference**

Mass of pulley \_\_\_\_\_  $m_{\text{equiv}}$  for pulley \_\_\_\_\_  
 Mass of string \_\_\_\_\_ Mass of hangers \_\_\_\_\_

Data

Ascending mass,	$m_1$							
Descending mass,	$m_2$							
Distance	$\Delta y$							
Time, $t$	1							
	2							
	3							
Time	Avg.							

Calculations

Calculated acceleration, $a_{\text{exp}}$								
Total mass $m_1 + m_2 + m_{\text{equiv}}$								
Mass Difference $(m_2 - m_1)$								
Net Force $(m_2 - m_1)g$								
Theoretical Acceleration, $a_{\text{theo}}$ .								
% Diff between exp. and theo.								

Calculations (show and explain work)

**Vary the net force (keep the total mass constant at 260 grams)**

Mass of pulley \_\_\_\_\_  $m_{\text{equiv}}$  for pulley \_\_\_\_\_  
 Mass of string \_\_\_\_\_ Mass of hangers \_\_\_\_\_

Data

Ascending mass,	$m_1$							
Descending mass,	$m_2$							
Distance	$\Delta y$							
Time, $t$	1							
	2							
	3							
Time	Avg.							

Calculations

Calculated acceleration, $a_{\text{exp}}$							
Total mass $m_1 + m_2 + m_{\text{equiv}}$							
Mass Difference $(m_2 - m_1)$							
Net Force $(m_2 - m_1)g$							
Theoretical Acceleration, $a_{\text{theo.}}$							
% Diff between exp. and theo.							

From graph:

Slope \_\_\_\_\_ units  $g$  from graph \_\_\_\_\_ units                       
theoretical g      units

Intercept \_\_\_\_\_ units  $f$  from graph \_\_\_\_\_ units

Calculations (show and explain work)

**Vary the net force (keep the total mass constant at 360 grams)**

Data

Ascending mass,	$m_1$							
Descending mass,	$m_2$							
Distance	$\Delta y$							
Time, $t$	1							
	2							
	3							
Time	Avg.							

Calculations

Calculated acceleration, $a_{exp}$								
Total mass $m_1 + m_2 + m_{equiv}$								
Mass Difference $(m_2 - m_1)$								
Net Force $(m_2 - m_1)g$								
Theoretical Acceleration, $a_{theo.}$								
% Diff between exp. and theo.								

From graph:

Slope \_\_\_\_\_ units  $g$  from graph \_\_\_\_\_ units   
theoretical g units

Intercept \_\_\_\_\_ units  $f$  from graph \_\_\_\_\_ units

Calculations (show work)