Centripetal Force: The center-seeking force (8/22/05)

Introduction

When an object is moving in a circle at constant speed the acceleration is *not* zero. Even though the speed may be constant the *velocity* is changing. Remember that the acceleration is the rate of change of velocity, and velocity is a vector with both a magnitude (speed) and a direction. Thus to move in a circle a force, **F**=m**a** is required. This force must point towards the center since when moving in a circle the direction of the velocity is continually "bending" towards the center. It can be shown that, for circular motion at constant speed, the magnitude of the acceleration is a = v^2/R. Therefore, the centripetal force must be F = mv^2/R, where v is the speed, and R is the radius of the circle. In this lab we will attempt to verify this relation.

Equipment

- centripetal force apparatus
- mass set
- mass hanger
- stopwatch
- mass balance
- pliers
- level
- ruler
- string & scissors (for repairs)

Theory

In the absence of any external force, Newton's first law tells us that any object will either remain at rest or continue to move *in a straight line* at constant velocity. When moving in a circle, therefore, a force is required to change the *direction* of motion *even if the speed does not change*. Since the *speed* does not change the force must be directed perpendicular to the direction of motion (i.e. toward the center of the circle). Whatever its source (gravity, a rope, a spring) we call this type of force a *centripetal force*, meaning center-seeking. We can see that this force is necessary for circular motion by imagining what would happen if we removed it. When a mass tied to a rope is twirled in a circle, the inward force of the rope keeps it moving in a circle. What would happen to the mass if we cut the rope? You might think it would fly outward, perpendicular to the circle. In fact its inertia would keep it moving in a straight line with the same velocity it had at the instant the rope was cut. In other words, it would continue to move along a straight line tangent to the circle (see dashed arrow in Fig. 1). The natural condition of the free mass is to travel in a straight line. We must apply a "centripetal force", by means of the tension in the rope, to cause the direction of the velocity vector to change if we want it to travel in a circle.

Circular Motion:

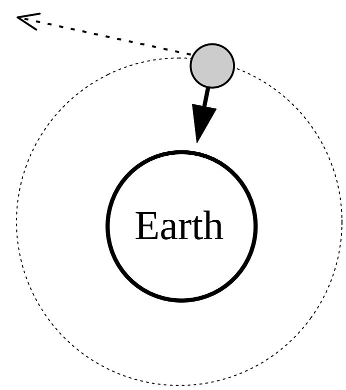


Figure 1: The moon travels in a nearly circular orbit about the Earth due to the inward force of gravity. Without this force it would travel in a straight line

When traveling in a circle of radius *R* a distance Δs = $2\pi R$ is covered every revolution. The period (time for one revolution) is denoted by the symbol *T*. The average speed, *v*, is thus give by

$$v = \frac{\Delta s}{\Delta t} = \frac{2\pi R}{T} \tag{1}$$

For uniform circular motion (i.e. at constant speed), it can be shown that the inwardly directed centripetal acceleration must be

$$a = \frac{v^2}{R} = \frac{1}{R}\frac{4\pi^2 R^2}{T^2} = \frac{4\pi^2 R}{T^2} \tag{2}$$

From Newton's 2nd Law, the net (total) force on the object must therefore be

$$F = ma = \frac{mv^2}{R} = \frac{m4\pi^2 R}{T^2}. \quad (3)$$

This force is directed towards the center of the circle.

Procedure

A diagram of a centripetal force apparatus is shown in Figure 2. The inward force is provided by a spring and the hanging bob may be set in circular motion by manually twirling the rotational shaft. In this lab you will measure the force of the spring for different radii, then you will measure the period of the circular motion required to keep the bob moving in a circle for each radius. If the motion is too slow, the spring will pull the bob closer inward (toward the center). If the motion is too fast, the force of the spring will be insufficient to keep the bob moving in a circle at that radius, and it will travel in a circle of larger radius.

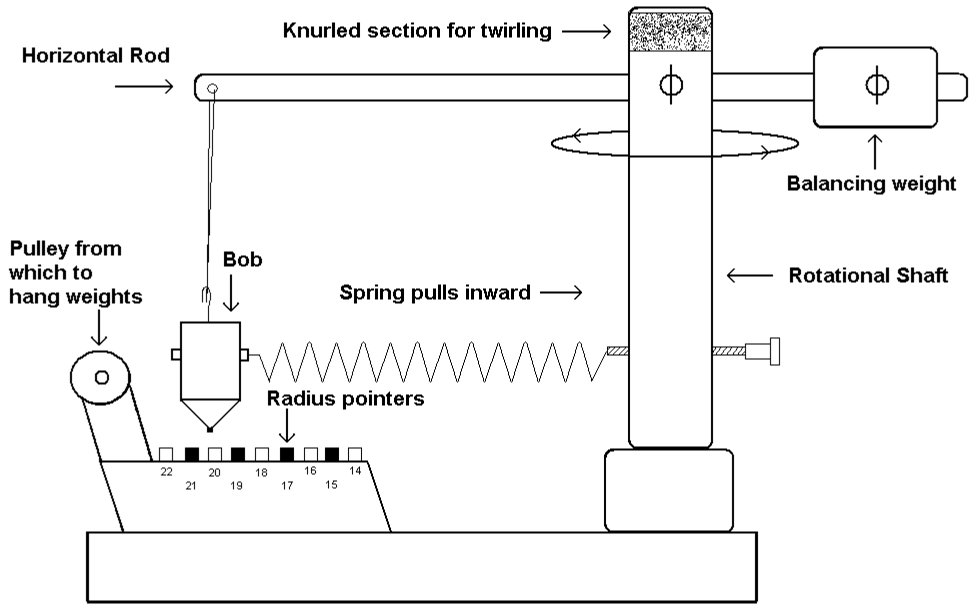


Figure 2: Centripetal Force Apparatus

<u>Measuring the force of the spring for a given radius.</u>

Use the balance to determine the mass, m_B, of the bob, and record it in Table One.

Adjust the position of the horizontal rod until the bob is hanging directly over the 14 cm location of the radius pointer <u>with no spring attached</u>. Record this radius in Table One. Pull the bob inwards and attach it to the spring. Attach a string with a hook to the other side of the bob. Pass the string over the pulley and attach a mass hanger to the string. Add masses to the hanger until the bob once again hangs straight over the 14 cm location. **(Question: Why is it important for the string and the spring to be horizontal and collinear?)** Record this mass, m_H, in Table One and calculate and record the weights of the hanging masses.

<u>Draw two Force Diagrams</u> Draw separate diagrams for the bob and for the hanging mass.

- The vertical forces on the bob are the tension in the vertical string, T_V, and the weight of the bob, W_B.
- The horizontal forces on the bob are applied by the spring, call this F_B, and the tension in the string that passes over the pulley to the hanging mass, call this T_H.
- The hanging mass has only vertical forces on it: T_H up, and W_H down. Since the mass is not accelerating, these forces must add to zero, and are therefore of equal magnitude.
- The horizontal forces on the bob are T_H from the string, outward, and F_S from the spring, inward (i.e. towards the rotational shaft). Since the bob is not accelerating, these forces must also be equal and opposite. Therefore, the force of the spring, $F_S = T_H = W_H = m_H g$.

Measuring the period of circular motion

Remove the hanging masses and their string. The spring now pulls the bob towards the center. Now, manually twirl the bob using the knurled section at the top of the rotational shaft until the bob hangs vertically over the 14 cm location. (NOTE: If your apparatus wobbles during rotation you may need to adjust the balancing weight for better dynamic balance.) Notice that if you twirl slowly the bob is pulled closer inward by the spring and if you twirl too fast the bob travels outwards towards a larger radius. At just the right speed the force of the spring will be equal to the centripetal force required to keep the bob moving at constant radius. Practice twirling the rod until you can keep it moving at a nearly constant speed with the bob hanging vertically directly over the desired location (i.e. radius).

In order to determine the period of rotation you must keep the bob twirling at constant speed as you time and count some number, N, of revolutions. Use a stopwatch to measure the total time for N revolutions and the period (time for one revolution) can be calculated by dividing the total time by the number of revolutions (N). While the timing and counting the number of revolutions, rotation speed is maintained constant by periodic twirling of the knurled section on the rod.

To measure the period have one lab partner (the twirler) turn the rod while another (the timer) holds the stop watch and a third (the counter) counts the number of times the bob passes over the pointer. When the twirler feels that he is maintaining a constant speed which keeps the bob centered over the pointer, he signals the timer and counter by saying "three, two, one, ***go***" After the timing starts the counter counts the number of times the bob passes over the pointer. After counting 50 to 100 turns, the counter signals the end of the count, for example by saying "67, 68, 69, **stop**" and the timer stops the watch. Record the total number of revolutions, N, and the time, T_{total} in Table Two. The period measured is then $T = T_{total}/N$.

Repeat for three additional trials, having lab partners switch roles (twirler, timer, counters). Record all trials and calculate an average period for each radius and bob mass.

Calculations and comparison

Calculate the velocity that corresponds to this period and radius.

Calculate the centripetal force required for this speed and radius. Compare this to the force applied by the spring and calculate a percentage difference.

Repeat for different bob masses

Loosen the knurled nut on top of the bob and add a slotted mass (about 100 grams). Consider the force diagrams you drew previously. Is there any change in the force of the spring if the bob's mass changes?

Measure the period of rotation required to keep the bob moving at constant radius as described above. Repeat this measurement three times. Again compare the calculated centripetal force required for this mass to the force of the spring. How does the force change when the mass of the bob changes? How has the velocity changed?

Repeat for a different radius.

Remove the added mass from the bob. Move the horizontal rod until the bob hangs vertically over the 20 cm pointer location (with no spring attached). (NOTE: You may need to adjust the spring – attach screw to maintain an achievable spring tension.) Now repeat the measurement of the force of the stretched spring for this larger radius by re-attaching the string via the pulley to the hanging masses. Add masses until the bob once again hangs vertically. Record the masses and calculate the weight of the hanging masses. Record this data in Table One.

Once again remove the hanging masses and measure the period of rotation needed to keep the bob moving at constant speed and hanging vertically over the pointer. Record in Table Two.

Calculate the required centripetal force and compare to the measured force of the spring. How does the centripetal force change with radius?

Questions

1. When the bob is moving in a circle, what provides the centripetal force?
   If it is suddenly removed, what would happen to the Bob?

2. What does Newton's First Law predict would happen to the bob if all force were removed?
   Is this consistent with your answer to Question 1?

3. Examine equation (3). If everything in the equation were constant except that the mass increased, how would the centripetal force required to move in a circle change?
   If you double the mass what happens to the required centripetal force?

4. Examine equation (3). If everything in the equation were constant except that the speed increased, how would the centripetal force required to move in a circle change? If you double the speed what happens to the required centripetal force?

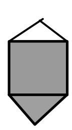
5. In your experiment, you changed the mass but kept the radius constant. Because the radius did not change, the force of the spring did not change. Instead, when you increased the mass, you had to change something else in order to make the required centripetal force stay the same. What did you change? Did you increase or decrease it? By how much (what ratio)? Is this what you would have predicted from equation (3)?

6. When you increased the radius of motion the force applied by the stretched spring increased. How did the velocity for circular motion change?

Force Diagrams for bob and for the hanging mass.

Draw and label vectors emanating from each object to depict the horizontal and vertical forces on them. Show that if the acceleration is zero (a=0) the force applied by the spring is equal to the weight of the hanging mass.

Bob



Mass

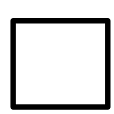


Table One: Calibration of the Spring Force

Experimental Data			Calculations	
Mass of Bob	Radius (Length from center of Bob to center of Rod)	Hanging masses, m_H	Hanging Weight, $m_H g$	Force of Spring for this Radius.

a) **Table Two: Measurement of period, calculation of required centripetal force.**

Bob Mass, m_B	Radius, R	Trial No.	N	Time T_{total}	Period T	Avg. T	$v = \frac{2\pi R}{T}$	$F = \frac{mv^2}{R}$	F_{spring} (from above)	% Diff.
		1								
		2								
		3								
		1								
		2								
		3								
		1								
		2								
		3								