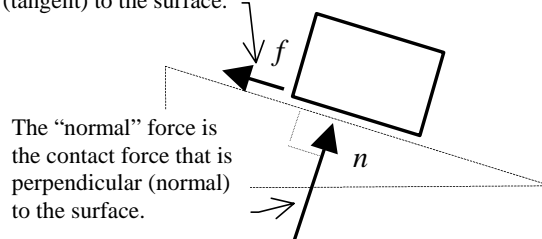


## Friction: Investigation of a model for friction (approximately 2hr. 15 min.) (11/7/06)

### Introduction

When two solid surfaces come in contact with each other there will be forces acting between the surfaces. These “contact forces” can be broken up into two components: a *normal force*, which is perpendicular to the surfaces and prevents the surfaces from penetrating each other, and a *friction force*, which acts parallel to the surfaces and prevents, or hinders, relative motion between the two surfaces (i.e. slippage). Both forces originate at the microscopic level in electromagnetic interactions between the atoms of the surfaces. Rather than attempting to understand all the details of this interaction, however, it is often useful to come up with a *model* for describing these forces. In this lab we will investigate a model for friction forces.

Friction is the contact force that is along (tangent) to the surface.



The “normal” force is the contact force that is perpendicular (normal) to the surface.

**Figure 1: Forces on a block resting on an inclined surface. Contact forces are broken into two perpendicular components: friction and the normal force.**

### Equipment

- adjustable, inclined plane with pulley
- wooden block with hook & string
- std. mass set & 50g hanger
- protractor
- graph paper (or Excel)
- supplemental mass set (400g)
- meter stick
- small rods & clamps
- beam balance
- long rod (tapered end)

### Before the Lab

Read the sections in your text describing friction. Read the section below and answer the questions.

### Theory

Friction is a contact force between two surfaces, which resists *relative* motion of the two surfaces: that is, it prevents or hinders the two surfaces from sliding relative to each other. The direction of the frictional forces is along, or tangent to, the surfaces, in a direction that will prevent or oppose relative motion. Experiment has established that friction depends on the materials of the two surfaces (their composition and roughness) and upon the *normal force* between the two surfaces. The normal force is the force of contact that is perpendicular to the surfaces, pressing them together. To a good approximation, for many surfaces, the frictional force is independent of the area of contact between the surfaces.

### Static Friction

When we attempt to make one object move relative to a surface with which it is in contact, a friction force will oppose the applied force. For example, suppose we apply a small force to push a book along a tabletop. As long as the applied force is not too great, a *static friction force* will oppose and prevent the occurrence of slippage. That is, the static friction force will be exactly equal to the applied force, but opposite in direction, resulting in no net force on the object. The amount of force needed to overcome the static friction force and cause the object to begin sliding along the surface

depends, of course, on the condition and material composition of both surfaces. It also depends on how much force is pressing them together. This force is perpendicular to both surfaces, and so is called the *normal force*, which we denote by  $n$ . As might be expected, the greater the normal force pressing the surfaces together, the greater is the amount of force needed to overcome static friction and cause sliding. Therefore, the simplest mathematical model for describing static friction assumes that the maximum force that may be applied to an object along a direction parallel to a contacting surface (i.e. “tangential” to the surface), without causing sliding, is directly proportional to the normal force. In other words, we can model it with the equation:

$$f_s \leq \mu_s n, \quad (\text{Equation 1: Static Friction})$$

where  $f_s$  is the static friction force,  $\mu_s$  is the *coefficient of static friction* (a number that depends on the material compositions and roughnesses of both surfaces), and  $n$  is the normal force pressing the two surfaces together. The static friction force is a reaction force that exactly cancels the net force applied to an object that would otherwise cause sliding along the surface. It is always less than or equal to the maximum value:  $\mu_s n$ . It’s zero when there is no net tangential force applied tending to make the two surfaces slide relative to each other. It increases as the applied force increases, exactly canceling it, until it exceeds the maximum,  $\mu_s n$ , whereupon the two surfaces break free and sliding begins.

### **Kinetic Friction**

Once sliding begins, the friction force stays relatively constant, independent of speed. Although it no longer prevents relative motion, it hinders or opposes this motion. This type of friction is called *kinetic friction* and is modeled by the equation:

$$f_k = \mu_k n \quad (\text{Equation 2: Kinetic Friction})$$

where  $f_k$  is the force of friction,  $\mu_k$  is the *coefficient of kinetic friction* and  $n$  is again the normal force. This equation shows that kinetic friction always has the same constant value any time the two surfaces are moving relative to each other (as long as the nature of the surfaces and the normal forces don’t change). The coefficient of kinetic friction is normally less than the coefficient of static friction, because once sliding begins the frictional force normally decreases slightly.

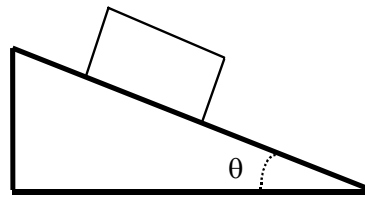
Note that these two equations only describe the *magnitudes* of the frictional forces. The *direction* is always along (i.e. parallel to) the surface, directed so as to prevent or oppose sliding.

### **Experimental Method A: The inclined plane**

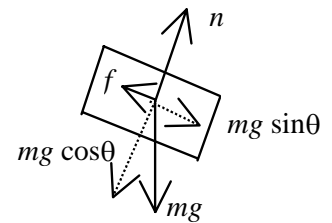
Figure 2 shows that for a block on an inclined plane the component of weight along the plane,  $mg \sin \theta$ , will act to accelerate the block down the plane. Friction opposes this motion by acting in the direction up the plane. If the angle is small enough or the coefficient of friction is large enough, friction can prevent sliding motion of the block. Once the maximum limit for static friction is exceeded, however, the block begins to slide down the plane. The block will accelerate as it slides down the plane, since the component of weight down the plane will be greater than the opposing force of kinetic friction.

Consider an inclined plane, as shown in Figure 2, whose angle,  $\theta$ , can be adjusted. If a block is placed on the plane while it is horizontal and the angle of the plane is slowly increased eventually the block will begin to slide. This is the point at which the limit of static friction has been reached.

Block on inclined plane:



Force diagram for the block.



**Figure 2: Block on an inclined plane**

We can write the forces on the block in terms of components along (tangent to) the plane and components perpendicular (normal) to the plane:

Perpendicular to the plane the acceleration is always zero and Newtons Laws tell us that

$$n - mg \cos \theta = ma_y = 0 \quad \text{or} \quad n = mg \cos \theta \quad (3)$$

Before the block slides, friction is less than or equal to the static limit and there is no acceleration along the plane either.

Thus the forces along the plane are given by:

$$mg \sin \theta - f_s = ma_x = 0 \quad \text{or} \quad f_s = mg \sin \theta \quad (\text{before block slides}) \quad (4)$$

In our model,  $f_s \leq \mu_s n$  and therefore

$$f_s \leq \mu_s mg \cos \theta \quad (\text{before block slides}) \quad (5)$$

The force of friction increases as the angle increases until the maximum value is reached. Exactly at this angle acceleration is still zero and:

$$f_s = mg \sin \theta_s = \mu_s mg \cos \theta_s \quad (\text{on the brink of sliding}) \quad (6)$$

From this we find that the angle at which the block slides,  $\theta_s$  is related to the coefficient of friction by the equation:

$$\tan \theta_s = \sin \theta_s / \cos \theta_s = \mu_s \quad (7)$$

This provides an experimental way of determining the coefficient of static friction.

In a similar way the inclined plane can be used to determine the coefficient of kinetic friction. Once the block begins to slide, the frictional force in our model is determined by the equation  $f_k = \mu_k n$ . For the block on the inclined plane, this becomes

$$f_k = \mu_k mg \cos \theta \quad (\text{as the block slides}) \quad (8)$$

Since the coefficient of kinetic friction is less than the coefficient of static friction, less force will be required to keep the block moving than was required to start the block moving. We can find an angle,  $\theta_k$ , at which the block can move at constant speed (no acceleration). At this angle the kinetic friction must exactly balance the weight down the incline. We will need to “push” the block to start it moving, but once it is started there will be zero net force down the incline as the block moves at constant speed. For this angle:

$$f_k = mg \sin \theta_k \quad (9)$$

and

$$f_k = \mu_k mg \cos \theta_k$$

therefore,

$$\tan \theta_k = \mu_k \quad (\text{as block slides at constant speed}). \quad (10)$$

This equation may be used to find the coefficient of kinetic friction. The block may need a “push” to get started but, once started, it must move at (nearly) constant speed.

## Experimental Method B: The block and pulley.

Another experimental arrangement for measuring the coefficients of friction is shown in Figure 3. Here the inclined plane is lowered to a horizontal position and force is applied to the block by a string running over a pulley and connected to a hanging mass. The force diagram shows the forces on both the block and the hanging mass. The tension,  $T$ ,

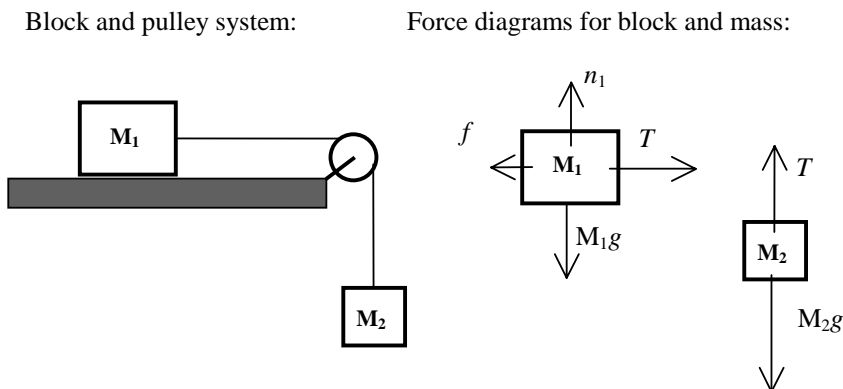


Figure 3: Block and pulley system

transmitted by the string is the same for both objects. (This assumes that the masses of the string and the pulley, as well as friction on the pulley, are all negligible.) We again break forces into components and find that, for the block  $M_1$ , the forces normal to the horizontal plane and along the plane are:

$$n_1 - M_1g = M_1a_{1y} = 0 \quad \text{and} \quad T - f = M_1a_{1x}. \quad (11)$$

For the hanging mass,  $M_2$  the forces are:

$$M_2g - T = M_2a_2. \quad (12)$$

The acceleration of the block perpendicular to the plane,  $a_{1y}$ , is always zero. The acceleration of the block along the plane will be equal to the downward acceleration of the hanging mass and so  $a_{1x}=a_2$ . If there is no acceleration of the block and the mass ( $a_{1x}=a_2=0$ ) then  $T=M_2g$  and at the angle for which the block just begins to slide (the limit for static friction):

$$f_s = T = M_2g = \mu_s n_1 = \mu_s M_1g \quad (\text{just as the block begins to slide}) \quad (13)$$

from which we determine that:

$$M_2 = \mu_s M_1 \quad (\text{just as the block begins to slide}) \quad (14)$$

This equation can be used to determine the coefficient of static friction by finding a mass,  $M_2$ , for which the block just begins to slide. Note that the basic relationship given by Equation (13) is a relationship between forces,  $M_2g$  and  $\mu_s M_1g$ , but because the factor of  $g$  is common to both, it cancels and the coefficient of friction is found in terms of a ratio of masses. A similar procedure can be used to find the coefficient of kinetic friction by finding a value for the hanging mass for which the block will slide at constant speed (after it has been given a “push” to overcome static friction). Again there is no acceleration so the forces exactly cancel and

$$f_k = T = M'_2g = \mu_k n_1 = \mu_k M_1g \quad (\text{block moves at constant speed}) \quad (15)$$

so that

$$M'_2 = \mu_k M_1 \quad (\text{block moves at constant speed}) \quad (16)$$

### Pre-lab questions

Read the introduction to the laboratory and then answer the following questions before beginning the lab.

The direction of a frictional force is always in a direction, which \_\_\_\_\_ relative sliding motion, or the tendency toward slippage, between two objects.

What are the two kinds of coefficients of friction called?

Describe the conditions under which each kind pertains.

Which kind generally has the greater magnitude?

Suppose a block of mass 25.0 kg rests on a horizontal plane, and the coefficient of static friction between the surfaces is 0.22. (a) What is the maximum possible static frictional force that could act on the block? \_\_\_\_\_ Newtons. (b) What is the actual static frictional force that acts on the block if an external force of 25.0 Newtons acts horizontally on the block? \_\_\_\_\_. Show your work. (Did you attach units to your answer? If not, it's meaningless.)

In experiments to measure the coefficient of kinetic friction by sliding a block down an inclined plane, it is required that the block move at constant velocity. Why is this a requirement?

Consider the following statement: Frictional forces on a block on an inclined plane are always directed down the plane. Is it: (a) True, or (b) False. Explain.

A 5.0 kg block rests on a horizontal plane. A force of 10 N applied horizontally keeps the block moving horizontally at constant velocity. What is the coefficient of kinetic friction between the block and the plane? ( $g = 9.8 \text{ m/s}^2$ ) Show your work.

What are the units of the coefficients of friction?

How do the coefficients of friction depend on the contact surface area between two objects?

How does the coefficient of kinetic friction depend upon the speed of a moving object?

### **Objectives of this lab**

In this experiment the nature of the frictional forces between a smooth wooden block and a smooth flat board will be investigated to check the validity of our models for friction forces. The board will be used in the horizontal position and as an inclined plane to accomplish the following objectives:

To determine  $\mu_s$  and  $\mu_k$  by two different methods

To test whether  $\mu_s > \mu_k$

To determine whether the coefficients of friction are independent of the normal force

### **Part A: Board as an Inclined Plane**

**Setup:** Insert the tapered end of a long rod vertically into the rod-holder hole on the countertop and attach a rod clamp to it. Then put a small rod horizontally into the rod clamp and insert it into the hole in the side of a ramp board. The angle of incline can be adjusted by raising or lowering the clamp on the vertical rod.

### **Procedure (A)**

#### **Determining the coefficient of static friction**

Place the block with one of its larger surfaces on the board. Adjust the angle of the board until the block just begins to slide on its own. Record the angle at which it slips in the appropriate Data Table (Static Friction, A) as  $\theta_s$ . (Note: This angle may be measured directly using a protractor or calculated using the inverse tangent function on your calculator and measurements made with your meter stick. Try both ways as a cross check! Which method do you think is more accurate? Why?) Repeat the procedure at least three more times, allowing each lab partner to try adjusting the board. Each time place the block in a slightly different place on the plane in order to average the effects of any irregularities in the surfaces. Note which side of the block and which side of the board is used, and continue to use the same surfaces for all the measurements made in this experiment.

#### **Determining the coefficient of kinetic friction**

Again place the block on the board, using the same surface as you used previously. This time you will incline the board until the block is able to move down the plane at constant speed after it is given a slight push to start its motion. It may take several attempts to find the angle for which you

can “nudge” the block to get it started and for which the block then moves at constant speed rather than accelerating. (The block may start and stall as it slides) Record the angle at which this occurs as  $\theta_k$ . Repeat this process three more times for a total of four trials. Again try to use different parts of the board in order to average the effects of irregularities in the surface of the board.

### **Calculations and Graphs (A)**

1. For each  $\theta_s$  calculate  $\tan \theta_s$  (see Equation 7) and record the values in the appropriate Data Table. Determine  $\mu_s$  for each of the four measurements, then, calculate the average value of  $\mu_s$  determined by this method. From the variations in your measurements make an estimate of uncertainty in your value of  $\mu_s$ .

For each  $\theta_k$  measured, calculate  $\tan \theta_k$  (see Equation 10) and record the values in the Data Table. Calculate and record the mean  $\mu_k$  for the four measurements. Make an estimate of uncertainty.

## **Part B: Horizontal Board with Pulley**

### **Procedure (B)**

#### Determining the coefficient of static friction

1. Determine the mass of the block using the balance. Record it in the Data Table (Static Friction, B) as  $M_1$  in the space labeled “0 kg added”.
2. Place the board in a horizontal position on the laboratory table with pulley beyond the edge of the table as shown in Figure 3. Place the block on the board using the same surfaces as you used previously.
3. Attach a piece of string to the hook in the block. Place it over the pulley and attach the mass hanger to the other end of the string. **Adjust the position of the pulley until the string pulls parallel to the horizontal surface.** Carefully add mass to the mass hanger to find the minimum mass needed to cause the block to move. Record the value of the hanging mass as  $M_2$  in the Data Table (Static Friction, B). (Should the mass of the hanger be included in  $M_2$ ?) Repeat the procedure two more times for a total of three trials using different parts of the board each time.
4. Repeat step 3 but add 0.2 kg to the top of the block. Record the value of the mass of the block plus 0.2 kg as  $M_1$ . Again determine the minimum mass needed to just cause the mass  $M_1$  to move. Do three trials and record each as  $M_2$  in the Data Table (Static Friction, B).
5. Repeat step 4 but with 0.4 kg added to the top of the block.

#### Determining the coefficient of kinetic friction

Perform a similar set of measurements as just described, but this time determine the masses,  $M_2$ , needed to keep the block moving at constant velocity after it has been started with a small push. Again take three trials for each case and use values of  $M_1$  beginning with the mass of the block and incrementing in steps of 0.2 kg up to a total of 0.4 kg of added mass. Record the values of  $M_2$  and  $M_1$  for all the cases in the Data Table (Kinetic Friction, B).

### **Calculations and Graphs (B)**

Calculate the mean (average)  $M_2$  of the three trials made for each of the values of  $M_1$  used in part B.

Do this for both the static and kinetic friction cases. Record these values in the appropriate Data Table.

According to Equation (14) there is a linear relationship between  $M_2$  and  $M_1$ , provided that  $\mu_s$  is a constant. Graph the static friction data for the horizontal plane case using  $M_2$  as the ordinate (vertical axis variable) and  $M_1$  as the abscissa (horizontal axis variable). If  $\mu_s$  is constant this graph should be a straight line. Show on the graph the straight line obtained from the linear least squares fit. From the slope of the line determine  $\mu_s$ .

Graph the kinetic friction data for the horizontal plane case using  $M_1$  as the abscissa and  $M_2$  as the ordinate. Show on the graph the straight line obtained from the linear least squares fit. Determine  $\mu_k$  from the slope of the line.

### Questions

1. Discuss the agreement between the two differently measured values of  $\mu_s$  determined in part A and part B. Calculate the % difference for these two methods using the following:

$$\% \text{ difference of quantity "x"} = \frac{x_1 - x_2}{\text{average x}} \times 100\% . \quad (16)$$

2. Provide a similar discussion for the kinetic friction data.
3. In part B, if the coefficient of friction depended on the mass  $M_1$ , then the equation  $M_2 = \mu M_1$  would not be a straight line. Does it appear as if  $\mu$  is independent of  $M_1$ ?
4. For which method (A or B) did you measure values of  $\mu_s$  and  $\mu_k$  for *different* values of the normal force. (Hint: examine equations (3) and (11) to find the normal force. Assume that any variations in angle for part A were due to experimental uncertainty.)
5. To what extent do your data confirm the expectation that the coefficients of friction, both static and kinetic, are independent of the normal force? State the evidence for your opinion.
6. Do your data confirm the expected relationship between  $\mu_s$  and  $\mu_k$ ? State clearly what is expected and what your data indicate.



Note: If you try different types of surfaces, you can record that under "Surface Used". For example: "rough", "smooth", "large side", "small side".

Mass of hanger=\_\_\_\_\_

**Data Table: Static Friction, B**

Data			Calc. Average $M_2$ for this $M_1$
Mass added	$M_1$	$M_2$	
0 kg			

**Data Table: Kinetic Friction, B**

Data			Calc. Average $M_2$ for this $M_1$
Mass added	$M_1$	$M_2$	
0 kg			

What is the relationship between the normal force and  $M_1$ ? (See equation 11)