Balancing Torques and the Center of Gravity (9/13/04)

Introduction

Just as force is required to accelerate a mass, a *torque* is required to produce angular acceleration. A torque is a force applied at a distance offset from some axis of rotation. This distance is often called the *lever arm* and the larger the lever arm is the more rotational acceleration can be produced by the force. The torque, τ, is defined by

$$\tau = r_{\perp} \times F$$

where $r_{\perp}$ is the *lever arm:* the distance from the axis of rotation to the line of the applied force, as shown in Figure 1. For rotation in two dimensions, torques can cause counter-clockwise (positive) rotation or clockwise (negative) rotation.

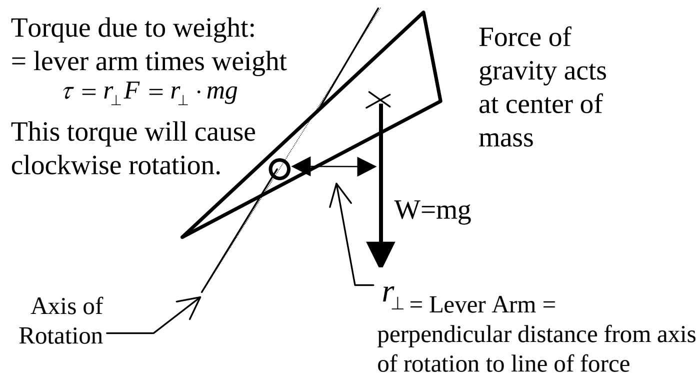

Figure 1: The weight of an object can cause a torque about an off-center axis

Equipment

- meter stick
- pivot
- 3 meter-stick clamps
- 1 unknown mass
- 5g mass hangers
- slotted mass set
- balance

Theory

In this lab you will study the special case of an object in *static equilibrium*: it is neither accelerating nor rotating. This means that there is both no net force and no net torque on the object. Any forces and torques applied to the object must be balanced. For example any torque which would cause clockwise rotation must be balanced by a torque which would cause counter-clockwise rotation.

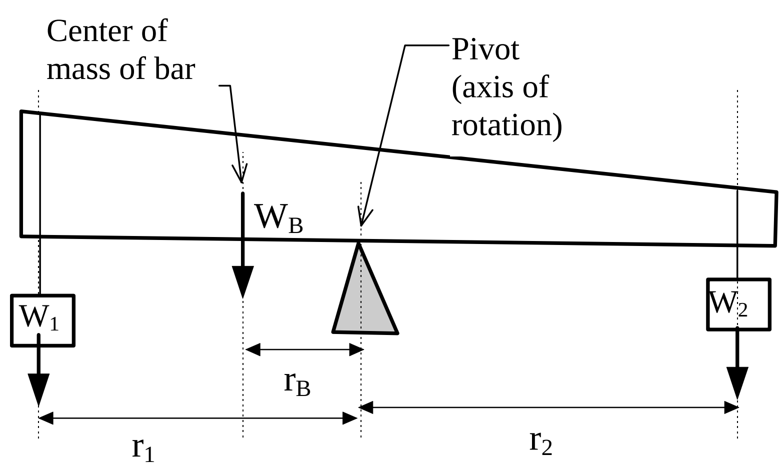

Figure 2: Objects balanced on a pivot: there is no net force and also no net torque. The weight acts at the center of mass.

One of the external forces on the object might be its weight. Although the force of gravity acts on every point throughout the object, the net effect is the same as if all the weight were concentrated at the *center of mass.* We say that the *weight acts at the center of mass,* which is sometimes also called the *center of gravity*.

Figure 2 shows a bar balanced on a pivot. If the bar were to rotate then the pivot would be the axis of rotation. In order for it not to rotate the torques produced by all the forces must balance. The weight, $W_B = m_B g$, of the bar itself acts at the center of mass of the bar. The lever arm for this force is shown as r_B so the torque it would produce is $\tau = r_B W_B$ which would tend to make the bar rotate counter clockwise. Similarly the hanging mass M_1 also produces a positive counter-clockwise torque , $r_1 W_1$, while the second hanging mass, M_2, produces a negative, clockwise torque, $- r_2 W_2$. For the bar to be balanced the net torque must be zero:

$\tau_{total} = r_B W_B + r_1 W_1 - r_2 W_2 = 0$

or

$$\tau_{CCW} = \tau_{CW} \quad \text{so that} \quad r_B W_B + r_1 W_1 = r_2 W_2$$

Procedure

A meter stick balanced on a pivot will be used to investigate how balancing torques create an equilibrium.

<u>A. Meter stick supported at center of gravity:</u>

Determine the weight of your meter stick and the three clamps you will use (one for the center and one on each end). Record this data.

Adjust the position of the meter stick in the center clamp until you can balance it on the pivot. Balance the meter stick on the pivot by moving it inside the center clamp until you reach an equilibrium position. The center of mass should be over the pivot now. Record this position.

Two known weights (Do not use equal weights!)

Select two unequal weights and attach them to either side of the stick by strings or by attaching them to clamps, in which case the weight of the clamp is included in the total weight. Adjust their positions until you find a balance. Record the weights used, (W), and their positions as read on the meter stick (X). Make a sketch of the arrangement and label the positions of the weights and the center of mass.

Calculate the lever arm for each weight. Indicate this distance on your sketch as well.

Calculate the net clockwise torque and the net counterclockwise torque. Find the percent difference between the two. How does this compare to the condition required for balance?

As you move the two weights how is the rotational equilibrium disturbed? Is there more than one way to position the two weights to achieve balance? Vary the positions of the weights until you have determined a general condition for balancing the weights. Record your observations and conclusions.

In the second data table record the weights and positions corresponding to your new equilibrium. Calculate the CCW and CW torques and their percent difference.

Three known weights (not all the same)

Repeat using three weights (at least two different) at three different positions. Find an equilibrium. Sketch your configuration and record your weights and positions. Compute clockwise and counter-clockwise torques and their percent difference.

Now choose two different positions for the first two weights. Calculate the lever arm required for the third weight to balance these two. Record your prediction then experimentally find the actual position required. Calculate the percent difference between the predicted and measured positions.

Unknown Mass:

Choose a lead weight to use as an unknown mass. With the meter stick balanced at its center of mass, hang the unknown mass on one side. On the other side place a known mass and adjust its position until you find balance. Describe the calculations needed to find the unknown mass and record your results. (Don't forget to include the effects of any clamps or mass hangers you use.)

Weigh your mass with the triple-beam balance and compare the result to your calculation.

B. Meter stick supported away from center of gravity:

With only one weight suspended from one end of the meter stick, move the center support away from the center of the meter stick until you find a rotational equilibrium.

Calculate the torque about the pivot due to the single mass hanging from the stick.

The single weight is being balanced by the weight of the meter stick, which acts as if it were all concentrated at the center. Pretend that the meter stick is in fact massless. Where would you need to hang a second mass, equal to that of the meter stick, so that its lever arm would be sufficient to balance the single weight? Calculate this lever arm (from the new position of the pivot). Compare its position to the position of the center of mass.

Questions

When two unequal masses are balanced on either side of the meter stick, what is the relationship between the ratios of the masses and the ratios of their lever arms?

Is it possible that there is no net (total) force on an object (such as the meter stick) but a non-zero torque? Explain and give and example.

How does the triple-beam balance work?

A. Meter stick supported at center of gravity:

Mass of stick:		
Mass of left clamp: Weight=	Mass of center clamp Weight=	Mass of right clamp Weight=:
Position of Center of mass of stick:		

Two Known Weights Data Table (first equilibrium arrangement of weights):

Sketch (labeled)	Weights, Position		Lever Arms:	Results:
	W_1= W_2=	X_1 = X_2 =	r_1= r_2=	τ_{CCW}= τ_{CW}= % Diff:

Discussion of how the balance of the meter stick varies as the weights are moved:

Two Known Weights Data Table (alternative equilibrium arrangement of weights):

Sketch (labeled)	Weights, Position		Lever Arms:	Results:
	W_1= W_2=	X_1 = X_2 =	r_1= r_2=	τ_{CCW}= τ_{CW}= % Diff:

Additional Discussion:

Three Known Weights Data Table (first equilibrium arrangement of weights):

<table>
<tr><td rowspan="3">Sketch (labeled)</td><td colspan="2">Weights, Position</td><td>Lever Arms:</td><td>Results (include direction)</td></tr>
<tr><td colspan="2">W_1= X_1 =
W_2= X_2 =
W_3= X_3 =</td><td>r_1=
r_2=
r_3=</td><td>τ=
τ=
τ=</td></tr>
<tr><td>τ_{CCW}=</td><td colspan="2">τ_{CW}=</td><td>% Diff:</td></tr>
</table>

Three Known Weights Data Table (second equilibrium arrangement of weights):

<table>
<tr><td rowspan="4">Sketch (labeled)</td><td colspan="2">Weights, Position</td><td colspan="2">Lever Arms:</td><td>Results:</td></tr>
<tr><td colspan="2">W_1= X_1 =
W_2= X_2 =</td><td colspan="2">r_1=
r_2=</td><td>τ=
τ=</td></tr>
<tr><td>W_3=</td><td colspan="4">Predicted position:</td></tr>
<tr><td colspan="3">Measured position:</td><td colspan="2">% Diff:</td></tr>
</table>

Discussion and Observations:

Unknown Mass:

Sketch (labeled)	Weights, Position	Results:
	W_1= ??? X_1 = W_2= X_2 =	W_1 (calculated) W_1 (measured)

Describe Calculations needed to determine unknown mass (here or on additional page)

B. Meter stick supported away from center of gravity:

Mass of stick (previously measured)
Position of Center of mass of stick (previously measured)
Position of pivot:

Sketch (labeled)	Weight, Position	Lever Arm: (from pivot)	Torque:
	W_1= X_1 =	r_1=	τ=
	Lever arm required for weight equal to stick to balance single weight:	Position required for weight equal to stick:	
Compare predicted position to center of mass			

Calculations and Discussion: