Proof Study:
Gender Differences

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Prove the following: when you add any two odd numbers, your answer is always even. If you just said to yourself, “That’s easy,” then you most likely have had some mathematical background. But how much of a background in math does a person need to know how to prove this? Could a high school math student prove it? Maybe. How about a first year math major in college? Yeah, sure. Well then, what do you think about a high level math major in college? Yes, definitely. If that is what you are thinking, you’d be surprised!

Recently, as a group of senior math majors, we constructed a survey to test other student’s knowledge and competency when it comes to mathematical proof. The survey was administered to 44 high-level Rowan University math majors (juniors, seniors, and one sophomore). This breaks down to 17 males and 27 females. Each student was given fifteen minutes to complete the eight-question survey. The following contains the results of our survey focusing on the gender differences in the answers.

First, both of the male and the female grade point averages were calculated to determine whether or not that might be a factor in the different sexes’ answers. The average female GPA was approximately 3.21, and the average male GPA was approximately 3.19. We concluded that the difference isn’t large enough to have had an impact on each sex’s answers as a group. The students were also asked to check off on a list all the upper level math courses that he or she has taken. It was thought that the courses taken could have had an impact on the answers as well. There was a difference in the amount of males verses the amount of females that have taken real analysis. Real analysis is thought by our group to be almost a foundation course for the knowledge of proof. It is one of the first courses in which proof is the main focus of study. Only

52.9% of the males have taken the course whereas 74.1% of the females have taken real analysis. (All percentages are approximations evaluated to the nearest tenth). Other differences in the courses include; complex analysis and real analysis II which were each taken by 11.8% of the males and 7.4% of the females, probability and statistics which was taken by 64.7% of the males and 85.2% of the females, probability and statistics II which was taken by 11.1% of the females and none of the males, and applications of mathematics which was taken by 35.3% of the males and only 7.4% of the females. The rest of the courses showed little difference in the percentages.

With that said, now we can better analyze the results found in the questions on proof. The first area we will consider is our fellow math majors' understanding of proof. The students were asked to write a definition of proof. Out of the 44 students surveyed, two students, both female, failed to give any definition. Out of the 25 females that did give a definition, the common definitions included, 'an explanation of solving a problem, a process using theorems to prove that a statement is true or false, and a logical way to prove a theorem by using known axioms.'

The males, however, our group felt gave more in depth, thought out definitions than the females did. The males in general defined proof as an organized format using theorems, axioms, and techniques to prove something is true. Also, all 17 males gave a definition. Some of the male's definitions included; deriving a theorem from a set of axioms or established theorems in a formal system, a statement that holds true for any example and not simply examples of cases that work with actual numbers, and giving undisputable facts to show a theorem is true or false. The one sophomore who took the survey was a male who has taken real analysis. He defined proof as irrefutable evidence of truth. We felt that the definition was short and sweet, but not incorrect. The results were also compared between the juniors and seniors in each sex. However, no significant differences were found. Overall, the results showed that the males gave more distinct definitions of proof. We felt they answered the question better, even though not as many males took the real analysis course.

The next area we will consider is the students' recognition of proof. The following table[1] was on our survey:

Arthur, Bonnie, Ceri, Duncan, Eric, and Yvonne were trying to prove whether the following statement s true or false:

When you add any 2 even numbers, your answer is always even.

Arthur's answer

a is any whole number
b is any whole number
2a and 2b are any two even numbers
$2a + 2b = 2(a + b)$

So Arthur says it's true.

Bonnie's answer

2+2=4 4+2=6
2+4=6 4+4=8
2+6=8 4+6=10

So Bonnie says it's true.

Ceri's answer

Even numbers are numbers that can be divided by 2. When you add numbers with a common factor, 2 in this case, the answer will have the same common factor.

So Ceri says it's true.

Eric's answer

Let x = any whole number
y = any whole number
$x + y = z$
$z - x = y$
$z - y = x$
$z + z - (x+y) = x + y = 2z$

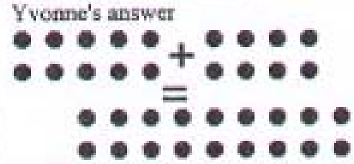
So Eric says it's true.

Duncan's answer

Even numbers end in 0, 2, 4, 6, or 8. When you add any two of these, the answer will still end in 0, 2, 4, 6, or 8.

So Duncan says it's true.

Yvonne's answer



So Yvonne says it's true.

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[1] This table was taken from L. Healy and C. Hoyles, *A Study of Proof Conceptions in Algebra*, Journal for Research in Mathematics Education **31** (2000), No. 4, pp. 396-428.

The students were asked to choose from the table, the "proof" that would be closest to the one they would write themselves. Arthur's answer came out on top for both sexes. 59.3% of the females and 58.8% of the males chose his answer. Ceri's answer was the second most popular, again for both sexes. 22.2% of the females and 23.5% of the males picked her answer. Bonnie's and Duncan's answers were each chosen by 4% of the females. The males had 11% choose Bonnie's answer and 6% choose Duncan's answer. None of these percentages are significantly different. The only difference in the answers was between the males' and females' choices for Eric's answer. Not one male chose Eric's answer, where as 11% of the females chose it.

Next, the students were asked to give a brief reason why they chose the proof they chose. The general consensus among the females who chose Arthur's answer as the best solution said it was because it showed the answer as any number multiplied by two which proves that it is even. Another popular reason among the females was because it would work for any number. The males gave answers that showed they felt Arthur's answer was simple, logical, and generalized. This doesn't show much of a difference between the two sexes' answers. However, for Ceri's answer, there was something of a difference. The females who chose this proof felt that it showed the definition of an even number. As for the males who chose Ceri's answer, one didn't even write anything as a reason why. Another chose Ceri's answer and when asked why, wrote, 'Arthur is using statements that are known to be true.' Perhaps this male was rushing through the survey! Recall that in the last question the biggest difference in the answers was that 11% of the females chose Eric's answer and none of the males chose it. The females chose that answer because it was an algebraic answer, showed different situations, and it was the most like a proof and not just an example. Hopefully someone reading this paper realized that Eric's proof is incorrect. It seems like the males did.

The students were also asked to choose the proof that they thought would receive the best grade from the teacher. 70.4% of the females chose Arthur's answer and 64.7% of the males chose Arthur's answer. This was the top choice again for both sexes. Also, 18.5% of the females and 35.3% of the males chose a different proof to receive the best grade than the one they would have done themselves. The majority of these votes were changed to Arthur's answer. Recall that none of the males chose Eric's answer for the proof most like the one they would do. However, 11.8% of them thought that Eric would receive the best grade. Perhaps they didn't realize that it was incorrect.

Next the students' knowledge of the validity of these proofs was tested. They were asked to circle all proofs they believed were valid. The proofs we consider to be valid are Arthur's, Ceri's, and Duncan's. Yvonne's we believe has the basis to be a valid proof, but needs something more. Since we were not sure ourselves, we didn't consider it valid or not valid. Bonnie's and Eric's proofs were considered not valid. 22.2% of the females and 29.4% of the males chose all three valid proofs. Out of those who chose all the valid proofs, 100% of the males chose only the valid proofs, whereas only 33.3% of those females chose only the valid proofs. The other 66.7% of the females chose Eric's incorrect proof as well. Those who chose Eric's and/or Bonnie's answer included 55.6% of the females and only 23.5% of the males. Most of the votes, however, were not for Bonnie's proof, but for Eric's proof. 44.4% of the females chose Eric's proof and 17.6% of the males chose it. Out of the students that voted for Eric's and/or Bonnie's answers, 71.4% of the females and 50% of the males didn't write anything when asked to do a simple proof. This implies that they may have been rushing to finish the survey.

Speaking of the simple proof the students were asked to write, recall from the beginning, 'when you add any two odd numbers, the answer is always even.' Our fellow high-level math majors were asked to write this proof. It must have been a little too complicated for the 25.9% of the females and 11.8% of the males who left the space blank! First we will consider what type of proof the students chose to do. The three formats that were used by the students were an algebraic format, a narrative format, and an empirical format. The most popular format used, an algebraic, follows along with Arthur's format. Arthur's answer was the one the students most popularly picked as the one they would write themselves. They were right. 66.7% of the females and 76.5% of the males wrote the proof algebraically. As for the narrative format, 3.7% of the females and 11.8% of the males chose to do the proof this way. Again, 3.7% of the females chose to do the proof in an empirical format when none of the males did. The chi square test showed no significant difference in this data.

Next, the validity of the proofs written by the students was analyzed. It was found that out of the 20 females who attempted the proof, 15% provided valid proofs of the wrong problem, 45% provided valid proofs for the right problem, 5% provided non valid proofs for the wrong problem, and 35% provided non valid proofs for the right problem. Out of the 15 males who attempted the proof, 13.3% provided valid proofs of the wrong problem, 46.7% provided valid proofs for the right problem, 20% provided non valid proofs for the wrong problem, and 20% provided non valid proofs for the right problem. Again, a chi square test determined that these percentages are not significantly different.

Over the entire survey, we feel that the males and females had equal proof construction, recognition, and understanding. A few calculated chi square tests help to show that the difference in the two sexes' answers may have been different at times, however, overall not significantly different. The results in general were a bit disappointing though. Only 36.4% of the high-level math majors surveyed at Rowan University could show that the sum of any two odd numbers is even!

References

1. E. Fennema et al., *A Longitudinal Study of Gender Differences in Young Children's Mathematical Thinking*, Educational Researcher 27 (1998), No. 5, pp. 6-11.
2. L. Healy and C. Hoyles, *A Study of Proof Conceptions in Algebra*, Journal for Research in Math Education 31 (2000), No. 4, pp. 396-428.