

Math Survey: Understanding Proof

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Introduction:

Math is a subject steeped in logic. Each branch of mathematics begins with a fundamental theorem and the blossom from there. Each new theorem leading to new mathematics and requires a proof in order to establish the validity of the theorem. Students of mathematics must be well versed in proof in order to succeed in this discipline. With this in mind, we conducted a survey on mathematical proof. We polled Rowan math majors on how they recognize, understand, and construct a mathematical proof.

The survey was constructed to analyze how the subjects define mathematical proof, and what style of mathematical proof they understand and hold to be valid. Finally we asked our subjects to construct a mathematical proof to understand how they employ their knowledge of mathematical proof.

The following are the results of our survey with respect to a general analysis of the subjects. No references are made to gender differences, however other variables such as grade level, GPA, and class loads are referenced. The results appear to be expected. We expect that Rowan math majors have a logical, procedural, algebraic understanding of mathematical proof as this is the most widely used in courses at Rowan.

Classifications for responses to the open ended questions were ranked in the following manner. For our definition of mathematical proof we grouped the responses into four apparent categories, truth, logical procedure, explanation, and a miscellaneous category. For the responses for the proof construction, proofs were grouped into type and validity. Type was defined as the three major types of proof, algebraic or formal, narrative, and empirical. Proofs were valid if they were constructed without error, regardless whether they give proof to the conjecture we posed for the subjects.

Proof Recognition:

Looking through the results for question 6, definition of mathematical proof, it appears that the majority of those surveyed chose proof to be defined as either truth or logical process. Out of the people who picked Arthur, 35.1% defined proof as a logical process and 32.4% claimed proof equals truth. The other dominant result in reference to proof recognition was the selection of Ceri's proof to question 7. From the students that chose Ceri, 37.9% had defined proof as a logical process, while 34.5% defined proofs as truth. Since Arthur and Ceri were chosen far and above the rest of the other proofs, the other results are closer together. Overall, 34.9% defined proof as truth, 31.1% said logical process, 12.3% defined it as explanation and reasoning, and 15.1% cited a mixture of 2 or 3 the other definitions. Also, 6.6% had no definition of proof.

In conclusion, it would seem to us that math majors at Rowan University have a steady knowledge of a definition of mathematical proof. There is very little difference between the number of students who chose logical process and those who selected truth to define proof. When you add in those who combined the two methods, it makes an even stronger

case for those definitions. Also, the fact that the two most selected and valid proofs, Arthur and Ceri, indicated truth and/or logical process as the reason behind the proof shows a unity in what Rowan University math majors recognize as a proof.

Proof Understanding:

In the matter of understanding what a mathematical proof is, Rowan math students are not fully competent. Their understanding of proof is based upon their response to question number six in our survey; "...write down your definition of proof." The responses were separated and grouped with similar responses. In grouping the students according to their answers, the largest number of students, 15 of the total 44, defined proof as something that showed the truth or validity of a statement. These students seem to be of upper level students having better than average GPA's ranging from 2.75 – 4.0's and most of which have already taken Intro to Real Analysis. The next largest group having 14 members defined a proof as a logical procedure. Only 10 of these students had already taken Intro to Real Analysis and their GPA's were almost identical to those of the first group. The third faction containing only 6 students believed that proof was evidence or an explanation of something. These students' GPA's were less than those of the previous groups and only half had taken Real Analysis. The fourth group of students that took this survey is considered the "other" category. These students answered in a way that didn't fit into the other three categories. Of these 6 students only 2 had Real Analysis and their GPA's were the lowest of all the groups.

Proof Construction:

35 out of 44 of the students who participated in the study attempted to answer question number 8. They were asked in this question to construct a mathematical proof for a given conjecture. Out of this group of 35, 21 students were able to construct valid proofs. However, only 16 out of these 21 students actually proved that when adding two odd numbers together the end result was always an even number.

According to these results, only thirty-six percent of the math majors who participated in this study were able to solve this fairly simple proof. These results may actually be worse than they appear. Considering that the question they were asked to prove was so similar to the one Arthur had already proven, it is very likely that some of these students simply used Arthur's example instead of figuring out the answer on their own.

Conclusions:

Our results show that Rowan math majors understand mathematical proof according to the number of upper level math classes they have taken, especially Intro to Real Analysis. The types of proof that the students are exposed to seemed to shape their definitions of proof. Consistently the choices for what the Teacher would grade the best and for their own choice of proof were the same. This parallel may suggest causation but further research would need to be carried out. Also the students' definition of proof tended towards truth and logic, these are two fundamental principles to a concrete and formal proof. This would be expected of a group of students who answer with algebraic proofs.

The few who expressed proof as an explanation show a higher level of understanding of proof that parallels the narrative type of proof. For truly understanding a proof means being able to explain the proof to another individual.

References:

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2. L. Healy and C. Hoyles, *A Study of Proof Conceptions in Algebra*, Journal for Research in Math Education 31 (2000), No. 4, pp. 396-428.