# Calculus II Review Sheet / Exam 2

1. Determine the convergence or divergence of the following. Explain the difference between the sequence and the series.

   a) $\left\{\frac{n}{2n+1}\right\}$ b) $\sum_{n=1}^{\infty} \frac{n}{2n+1}$

2. a) Show the sequence $\left\{\frac{n}{e^n}\right\}_{n=2}^{\infty}$ is monotone and bounded.

   b) Does the sequence converge? Justify your answer.

3. Consider the series $\sum_{K=1}^{\infty}\left(\frac{1}{K}-\frac{1}{K+1}\right)$. a) Find $S_1, S_2, S_3$, the first three terms in the sequence of partial sums.

   b) Find $S_n$, the $n^{th}$ partial sum of the series, in closed form and use it to find the sum of the series.

4. Determine whether the series converge or diverge. Name the test used and show all computations.

   a) $\sum_{K=1}^{\infty} \frac{2^K K^2}{K!}$ b) $\sum_{K=1}^{\infty} \frac{|\cos K|}{K^3}$ c) $\sum_{K=1}^{\infty} \frac{1}{2+3^K}$ d) $\sum_{K=1}^{\infty} K e^{-K^2}$

5. Determine whether the following converge absolutely, converge conditionally or diverge. Name the tests used and show all computations.

   a) $\sum_{K=2}^{\infty} \frac{(-1)^K 2^K}{\ln K}$ b) $\sum_{K=0}^{\infty} (-1)^K \left(\frac{6}{7}\right)^K$ c) $\sum_{K=1}^{\infty} \frac{(-1)^{K+1}}{3K+1}$

6. a) Show $\sum_{K=1}^{\infty} \frac{(-1)^K}{K^2}$ converges. b) If the sum of the series in a) is approximated by the $4^{th}$ partial sum, what can be said about the value of the error in the approximation?

7. a) Find the first four nonzero terms of the Taylor Series for $\sin x$ about $x=\pi$.

   b) Express the Taylor Series for $\sin x$ about $\pi$ in sigma notation.

8. Find the interval and radius of convergence for:

   a) $\sum_{K=1}^{\infty} \frac{(2x)^K}{3K}$ b) $\sum_{K=1}^{\infty} (-1)^{K+1} \frac{(x+3)^K}{K}$

9. Use the Maclaurin Series for $\frac{1}{1-x}$ to obtain the Maclaurin Series for $f(x) = \frac{x^2}{1+(2x)^4}$. For what values of $x$ does this converge?

10. Use series to approximate $\int_0^1 \sin(x^4)\,dx$ to 3-place decimal accuracy.