

Miscellaneous Pages

History of Mathematics

Dr Osler

H.M.
OSLER
□

History of Mathematics

Dr. Osler

Some interesting quotes:

Religion is the search for harmony with an unseen order.

William James

Mathematics is the science of order and mathematicians seek to identify instances of order and to formulate and understand concepts that enable us to perceive order in complicated situations.

Andrew M. Gleason and Shlomo Sternberg

The main duty of the historian of mathematics, as well as his fondest privilege, is to explain the humanity of mathematics, to illustrate its greatness, beauty and dignity, and to describe how the incessant efforts and accumulated genius of many generations have built up that magnificent monument, the object of our most legitimate pride as men, and of our wonder, humility, and thankfulness, as individuals. The study of the history of mathematics will not make better mathematicians but gentler ones, it will enrich their minds, mellow their hearts, and bring out their finer qualities.

—G. Sarton

HM
OSLER
2

HISTORY OF MATH
Dr T J Osler

MAJOR QUESTIONS FOR THE SEMESTER:

1. What are the major events in the history of mathematics. Define what you mean by a major event and select about five that you think are the most important.
2. In what environment (economic, political, religious, etc.) does creative mathematics flourish.
3. How do creative mathematicians view mathematics from a philosophical point of view. What motivates them to work in this area.

HM
OSLER
3

Cambridge
B2 IRP
USA
ralia

9

Cambridge

a

data

0

...mathematical ideas originate in empirics, although the genealogy is sometimes long and obscure. But, once they are so conceived, the subject begins to live a peculiar life of its own and is better compared to a creative one, governed by almost entirely aesthetical motivations, than to anything else and, in particular, to an empirical science. There is, however, a further point which, I believe, needs stressing. As a mathematical discipline travels far from its empirical source, or still more, if it is a second and third generation only indirectly inspired by ideas coming from 'reality', it is beset with very grave dangers. It becomes more and more purely aestheticising, more and more purely 'l'art pour l'art. This need not be bad, if the field is surrounded by correlated subjects, which still have closer empirical connections, or if the discipline is under the influence of men with an exceptionally well-developed taste. But there is a grave danger that the subject will develop along the line of least resistance, that the stream, so far from its source, will separate into a multitude of insignificant branches, and that the discipline will become a disorganised mass of details and complexities. In other words, at a great distance from its empirical source, or after much 'abstract' inbreeding, a mathematical subject is in danger of degeneration.

von Neumann (from the first paper in his collected works)

Some calculus tricks are quite easy. Some are enormously difficult. The fools who write the text books of advanced mathematics – and they are mostly clever fools – seldom take the trouble to show you how easy the easy calculations are. On the contrary, they seem to desire to impress you with their tremendous cleverness by going about it in the most difficult way.

Being myself a remarkably stupid fellow, I have had to unteach myself the difficulties, and now beg to present to my fellow fools the parts that are not hard. Master these thoroughly, and the rest will follow. What one fool can do, another can.

(from *Calculus Made Easy* by Sylvanus P. Thompson)

'Now,' Herbie says, 'wait a minute. A story goes with it.'

(from *A Story Goes With It* by Damon Runyon)

H.M. OSLER

WILLIAM BLAKE

THE SEER AND HIS VISIONS





Newton, color print, 1795

May God us keep
 From Single vision & Newton's sleep.

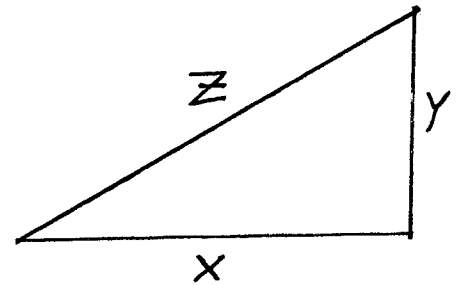
Blake's verse letter to Thomas Butts, Nov. 22, 1802, lines 87-88

37 God the Geometer (manuscript illustration)



HISTORY OF MATH
DR T J OSLER

FINDING PYTHAGOREAN
TRIPLES



$$X = 2MN$$

$$Y = M^2 - N^2$$

$$Z = M^2 + N^2$$

$$X^2 + Y^2 = Z^2$$

EACH PAIR OF
WHOLE NUMBERS
 $M \& N$

WITH $M > N$
GENERATES A
PYTHAGOREAN
TRIPLE VIA THE
ABOVE EQUATIONS,

M	N	X = 2*M*N	Y = M*M - N*N	Z = M*M + N*N
2	1	4	3	5
3	1	6	8	10
3	2	12	5	13
4	1	8	15	17
4	2	16	12	20
4	3	24	7	25
5	1	10	24	26
5	2	20	21	29
5	3	30	16	34
5	4	40	9	41
6	1	12	35	37
6	2	24	32	40
6	3	36	27	45
6	4	48	20	52
6	5	60	11	61
7	1	14	48	50
7	2	28	45	53
7	3	42	40	58
7	4	56	33	65
7	5	70	24	74
7	6	84	13	85
8	1	16	63	65
8	2	32	60	68
8	3	48	55	73
8	4	64	48	80
8	5	80	39	89
8	6	96	28	100
8	7	112	15	113
9	1	18	80	82
9	2	36	77	85
9	3	54	72	90
9	4	72	65	97
9	5	90	56	106
9	6	108	45	117
9	7	126	32	130
9	8	144	17	145
10	1	20	99	101
10	2	40	96	104
10	3	60	91	109
10	4	80	84	116
10	5	100	75	125
10	6	120	64	136
10	7	140	51	149
10	8	160	36	164
10	9	180	19	181

PERFECT NUMBERS

T J Osler Math Dept, Rowan University

Definition: A number n is “perfect” if it is the sum of all its divisors, except itself.

Example: The number 6 is perfect because $6 = 1 + 2 + 3$.

Theorem: If $2^n - 1$ is prime, then n is a prime number.

Proof: Suppose n is not prime and $n = rs$. Then

$$2^{rs} - 1 = (2^r)^s - 1 = (2^r - 1)(2^{r(s-1)} + 2^{r(s-2)} + \cdots + 2 + 1).$$

This last line shows that $2^n - 1$ can be factored, and thus it is not prime. So if n can be factored, so can $2^n - 1$, and the theorem is proved.

Definition: A if $2^p - 1$ is a prime number, it is called a Mersenne prime.

The largest known prime (found 11/04/01) is $2^{13466917} - 1$. It has 4,053,946 digits.

Theorem: If $2^p - 1$ is prime, then $n = 2^{p-1}(2^p - 1)$ is a perfect number.

Proof: The divisors of $n = 2^{p-1}(2^p - 1)$ are

$$1, 2, 2^2, 2^3, \dots, 2^{p-1},$$

$$(2^p - 1), 2(2^p - 1), 2^2(2^p - 1), 2^3(2^p - 1), \dots, 2^{p-1}(2^p - 1)$$

Summing these divisors (which include the number itself) we get

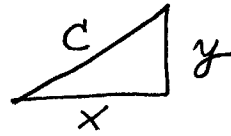
$$\begin{aligned} S &= 1 + 2 + 2^2 + 2^3 + \cdots + 2^{p-1} + \\ &\quad (2^p - 1) + 2(2^p - 1) + 2^2(2^p - 1) + 2^3(2^p - 1) + \cdots + 2^{p-1}(2^p - 1) \\ &= (1 + 2 + 2^2 + 2^3 + \cdots + 2^{p-1}) + \\ &\quad (2^{p-1} - 1)(1 + 2 + 2^2 + 2^3 + \cdots + 2^{p-1}) \\ &= (1 + 2 + 2^2 + 2^3 + \cdots + 2^{p-1})(1 + (2^p - 1)) \\ &= (1 + 2 + 2^2 + 2^3 + \cdots + 2^{p-1})(2^p). \end{aligned}$$

But, since $1 + x + x^2 + \cdots + x^m = \frac{x^{m+1} - 1}{x - 1}$, so $1 + 2 + 2^2 + 2^3 + \cdots + 2^{p-1} = 2^p - 1$.

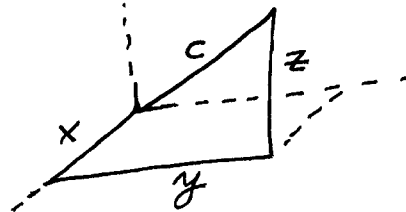
Therefore $S = 2^p(2^p - 1) = 2n$. However S includes the number n itself, so the sum of divisors of n not including n is $S - n = 2n - n = n$. Thus n is a perfect number, and the theorem is proved.

PYTHAGOREAN THM & GENERALIZATIONS

① $x^2 + y^2 = c^2$



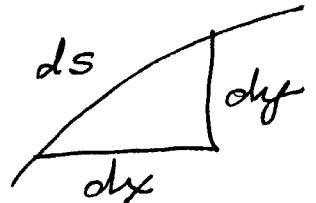
② 3 DIMENSIONS
 $x^2 + y^2 + z^2 = c^2$



③ n DIMENSIONS
 $x_1^2 + x_2^2 + \dots + x_n^2 = c^2$

④ DIFFERENTIAL ARC LENGTH

$$ds^2 = dx^2 + dy^2$$



⑤ HILBERT SPACE l_2

$$\vec{v} = (a_1, a_2, \dots)$$

$$\|\vec{v}\| = \sqrt{a_1^2 + a_2^2 + \dots}$$

↑
INFINITELY MANY a_n 's

⑥ L_2 NORM

$$\|f(x)\| = \sqrt{\int_a^b f(x)^2 dx}$$

⑦ RIEMANNIAN GEOMETRY

$$ds^2 = \sum_{i=1}^n \sum_{j=1}^n g_{ij} dx^i dx^j$$

⑧ SPACE-TIME INTERVAL

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$$

Handel: Ode for St. Cecilia's Day

HM

OSLER

9

New York Philharmonic, Leonard Bernstein, Conductor
Adele Addison, Soprano; John McCollum, Tenor
John Wummer, Flute; Laszlo Varga, Cello
Bruce Prince-Joseph, Organ
Rutgers University Choir, F. Austin Walter, Director

The practice of celebrating St. Cecilia's festival (November 22nd) with elaborate musical compositions was long established when Handel composed his first *Cecilia Ode* (the gorgeous *Alexander's Feast*) in 1736. As far back as 1683 a group of musical connoisseurs organized elaborate annual concerts in which the best poets and musicians participated. Poems in praise of the patron saint of music were written by Dryden, Shadwell, Congreve and others (in 1708 Pope turned out one of the finest of the species). These poems were set as ambitious choral-orchestral works by such composers as Purcell, Blow, Draghi, Eccles and Jeremiah Clarke. Purcell's *Cecilia Odes* are among his finest compositions, and yield place only to Handel's.

Handel's second *Ode for Saint Cecilia's Day* (1739), like *Alexander's Feast*, is a setting of words by John Dryden. By the third decade of the eighteenth century Dryden had become a classic. Many responsible critics considered him to be England's greatest poet—not barring Milton or Shakespeare. So it can well be imagined that Handel, in composing new music for Dryden's two odes (they had originally been set by Jeremiah Clarke and G. B. Draghi) was determined to put his best foot forward. And the two works are among the most stunning and "extravagant" he wrote—filled with inspired instrumental coloring, virtuoso word coloring, an often original approach to form. Inexplicably enough, Handel was not above lifting a tune here and there from other composers in the second *Ode*. The usual defense made for this is that music was considered common property in the baroque period. Nothing could be farther from the truth: plagiarism was as odious then as it is now. The simple proof is to be seen from the fact that Handel's enemies embarrassed him no end every time they found him out in a plagiarism. They might have had a field-day with the second *Ode*, which draws extensively on two harpsichord suites by a talented German contemporary of Handel's, Gottlieb Muffat (1690-1770). In Handel's defense, however, it can be urged that he never merely took over another man's work, but reworked it with characteristic and awesome craft so that it fitted perfectly his own needs. The grandiose fugue that brings the second *Ode* to a conclusion, for instance, is substantially the opening movement of one of Muffat's suites, transposed

from its monochromatic keyboard Ur-text to a veritable apotheosis for four-part chorus and full orchestra. For someone who knew the Muffat before he knew the Handel (the present writer found himself in that rather perverse situation) the effect is not unlike Cinderella before and after the touch of the magic wand.

I. OVERTURE

II. TENOR RECITATIVE

From harmony, from heavenly harmony

This universal frame began:

When Nature underneath a heap

Of jarring atoms lay,

And could not heave her head,

The tuneful voice was heard from high:

Arise, ye more than dead.

Then cold and hot and moist and dry

In order to their station's leap,

And Music's power obey.

(This virile accompanied recitative is filled with fine touches, but note especially the diminished sevenths in the orchestra depicting submerged Nature.)

III. CHORUS

From harmony, from heavenly harmony

This universal frame began,

From harmony to harmony

Through all the compass of the notes it ran,

The diapason closing full in Man.

(The expectation of "harmony," produced by the preceding tenor recitative is amply satisfied in this chorus. Handel sets strings and chorus running up and down the scale to illustrate harmony's pervasiveness, and the close is indeed a full diapason.)

IV. SOPRANO ARIA

What passion cannot Music raise and quell?

When Jubal struck the chorded shell,

His listening brethren stood around,

And, wondering, on their faces fell

To worship that celestial sound.

Less than a god they thought there could not dwell

Within the hollow of that shell,

That spoke so sweetly, and so well.

(Jubal's lyre, by a stroke of inspiration, is represented by a solo cello, singing a full-throated, romantic melody.)

V. TENOR ARIA AND CHORUS

The trumpet's loud clangour
Excites us to arms
With shrill notes of anger
And mortal alarms.

The double, double, double beat

Of the thundering drum

Cries, "Hark! the foes come;

Charge, charge, 'tis too late to retreat!"

(Handel dearly loved the combination of true kettledrum and tenor voice—viz. "Sound Alarm," from *Judas Maccabeus*. He surfer himself here. The naïve triple reference to "double beat" is a direct quote from Purcell's *King Arthur*, no plagiarism, but a graceful tribute to the earlier master.)

VI. MARCH

VII. SOPRANO ARIA

The soft complaining flute

In dying notes discovers

The woes of hopeless lovers,

Whose dirge is whispered by the warbling lute

("Flute" and "lute" are convenient rhyme words)

Dryden and excellent instrumental tips to Handel.

The two instruments combine to lend this air a languid melancholy.)

VIII. TENOR ARIA

Sharp violins proclaim

Their jealous pangs and desperation,

Fury, frantic indignation,

Depth of pains and height of passion,

For the fair, disdainful dame.

(A cocky tune, more suggestive of self-satisfaction than "jealous pangs and desperation." Handel has fun with the contrasted words, "depth" and "height.")

IX. SOPRANO ARIA

But oh! what art can teach,

What human voice can reach

The sacred organ's praise:

Notes inspiring holy love,

Notes that wing their heavenly ways

To mend the choirs above.

(St. Cecilia's own instrument was reputed to be the organ, and the praises of that instrument traditionally form the climax of compositions in her honor.)

X. SOPRANO ARIA

SIDE I OVERTURE: MINUET

FROM HARMONY; WHEN NATURE; FROM HARMONY

WHAT PASSION CANNOT MUSIC RAISE

THE TRUMPET'S LOUD CLANGOUR; MARCH

THE SELECTIONS (PUBLIC DOMAIN)

6:06

7:57

9:32

6:54

ARE FOLLOWED BY THEIR TIMINGS.

SIDE II THE SOFT COMPLAINING FLUTE

SHARP VIOLINS PROCLAIM

BUT OH! WHAT ART CAN TEACH

ORPHEUS COULD LEAD THE SAVAGE RACE;

OF SACRED LAYS

THIS COLUMBIA STEREO



FIDELITY RECORDING IS DESIGNED FOR USE ON 33 1/3 RPM

Library of Congress control card number D60-1381-200

ductor



MS 6206 Available on Regular—ML 5606

ector

nochromatic keyboard Ur-text to a synthesis for four-part chorus and full or someone who knew the Muffat before the Handel (the present writer found at rather perverse situation) the effect Cinderella before and after the touch wand.

IRE

RECITATIVE

ny, from heavenly harmony al frame began: re underneath a heap toms lay, ot heave her head, voice was heard from high: re than dead. nd hot and moist and dry their station's leap, s power obey.

accompanied recitative is filled with but note especially the diminished the orchestra depicting submerged

IS

ny, from heavenly harmony al frame began, ny to harmony the compass of the notes it ran, n closing full in Man. ation of "harmony," produced by the nor recitative is amply satisfied in this idel sets strings and chorus running n the scale to illustrate harmony's per- nd the close is indeed a full diapason.)

NO ARIA

on cannot Music raise and quell? struck the chorded shell, i brethren stood around, ring, on their faces fell that celestial sound. od they thought there could not dwell hollow of that shell, so sweetly, and so well. e, by a stroke of inspiration, is repre- a solo cello, singing a full-throated, lody.)

ARIA AND CHORUS

The trumpet's loud clangour Excites us to arms With shrill notes of anger And mortal alarms. The double, double, double beat Of the thundering drum Cries, "Hark! the foes come, Charge, charge, 'tis too late to retreat!" (Handel dearly loved the combination of trumpet, kettledrum and tenor voice — viz. "Sound an Alarm," from Judas Maccabeus. He surpasses himself here. The naïve triple reference to the "double beat" is a direct quote from Purcell's King Arthur, no plagiarism, but a graceful tribute to the earlier master.)

VI. MARCH

VII. SOPRANO ARIA The soft complaining flute In dying notes discovers The woes of hopeless lovers, Whose dirge is whispered by the warbling lute. ("Flute" and "lute" are convenient rhymes for Dryden and excellent instrumental tips to Handel. The two instruments combine to lend this aria its languid melancholy.)

VIII. TENOR ARIA

Sharp violins proclaim Their jealous pangs and desperation, Fury, frantic indignation, Depth of pains and height of passion, For the fair, disdainful dame. (A cocky tune, more suggestive of self-satisfaction than "jealous pangs and desperation." Handel has fun with the contrasted words, "depth" and "height.")

IX. SOPRANO ARIA

But oh! what art can teach, What human voice can reach The sacred organ's praise: Notes inspiring holy love, Notes that wing their heavenly ways To mend the choirs above. (St. Cecilia's own instrument was reputed to have been the organ, and the praises of that instrument traditionally form the climax of compositions in her honor.)

X. SOPRANO ARIA

Orpheus could lead the savage race, And trees unrooted left their place, Sequacious of the lyre, (This delicious aria, marked Alla hornpipe, features détaché playing from the violins. Handel proves the excellence of his latinity by his treatment of "sequacious.")

XI. SOPRANO RECITATIVE

But bright Cecilia raised the wonder higher, When to her organ vocal breath was given, An angel heard, and straight appeared, Mistaking earth for heaven.

XII. SOPRANO AND CHORUS

As from the power of sacred lays The spheres began to move, And sung the great Creator's praise To all the blest above, So when the last and dreadful hour This crumbling pageant shall devour, The trumpet shall be heard on high, The dead shall live, the living die, And music shall untune the sky. (The opening part of the final chorus is unique in Handel: one small soprano voice, entirely unaccompanied and obviously representing the angel mentioned in the preceding recitative, sings each line alone; then the entire chorus and orchestra repeat the verse after her. The joining of the trumpet with this soaring soprano toward the end is awesome, and the work closes fittingly with the magnificent fugue already referred to.)

David Johnson

Leonard Bernstein, the Philharmonic's Music Director since 1958, is a unique figure in the annals of American music. The profusion and the strength of his talents have made possible his many careers—as conductor, composer, pianist, teacher, lecturer, author.

Besides providing New York with brilliantly played music during the regular concert season, Bernstein and the Philharmonic have made three extensive tours: through Europe (including the Soviet Union, Poland and Yugoslavia) and the Near East, through South America and through the United States, from New Jersey to Hawaii—creating unprecedented good will by their inspired music-making.

THE SELECTIONS (PUBLIC DOMAIN) ARE FOLLOWED BY THEIR TIMINGS. SIDE II THE SOFT COMPLAINING FLUTE... 6:09 SHARP VIOLINS PROCLAIM... 5:38 BUT OH! WHAT ART CAN TEACH... 5:18 ORPHEUS COULD LEAD THE SAVAGE RACE; BUT BRIGHT CECILIA; AS FROM THE POWER OF SACRED LAYS... 12:22

DELITY RECORDING IS DESIGNED FOR USE ON 33 1/3 RPM STEREOPHONIC REPRODUCERS. 1

ANCIENT
ASTRONOMY TIMELINE

SOURCE HM OSLER
OCHOA & COREY
"TIMELINE BOOK OF
SCIENCE" 10

- 4200 BC EGYPTIANS INVENT 1ST KNOWN
CALENDAR 365 D, YR
12, 30 DAY MOS + 5 FESTIVAL DAYS
(MAY BE AS LATE AS 2700 BC)
-
- 2500 BC CONSTRUCTION OF STONEHENGE
(COMPLETED 1700 BC)
-
- 2296 BC 1ST COMET SIGHTING - CHINESE
-
- 1800 BC BABYLONIANS COMPILE ASTRO
DATA & STAR CATALOG
-
- 763 BC 1ST SOLAR ECLIPSE - BABYLONIANS
-
- 700 BC CHINESE RECORDS OF COMETS,
METEORS & METEORITES
-
- MAY 28
585 BC Thales of Miletus PREDICTS,
SOLAR ECLIPSE; Occurs during
battle of Lydians & Medes, -
they make peace
-
- 530 BC PYTHAGORAS
- EARTH IS SPHERE, - SUN, MOON,
STARS & 5 planets revolve
about earth in 8 concentric
spheres - Harmony of spheres
-
- 428 BC ANAXAGORAS DIES. He is imprisoned
for suggesting sun is a big hot stone
& NOT A GOD.
-
- 400 BC BABYLONIANS HAVE 12 SIGNS OF
ZODIAC
-
- 390 BC HERACLEIDES is born - argued
that MERCURY & VENUS orbit
sun,
-
- 352 BC 1ST SUPERNOVA - CHINESE
-
- 350 BC SHIN SHEN (Chinese) makes star
catalog with 800 stars

240 BC 1ST OBSERVATION OF HALLEY'S
COMET CHINESE

240 BC ERATOSTHENES FINDS DIAMETER
OF EARTH = 8000 MILES

200s BC ARISTARCHUS OF SAMOS
- 1ST TO ARGUE EARTH revolved about
SUN.
- EARTH rotates every 24 hrs

165 BC CHINESE OBSERVE SUNSPOTS

150 BC HIPPARCHUS OF NICAEA CALCULATES
DISTANCE FROM EARTH TO MOON

100's BC HIPPARCHUS compiles star catalog
- Discover precession of EQUINOXES
- ARGUES EARTH IS MOTIONLESS AT
CENTER OF UNIVERSE
- INVENTS SYSTEM OF STAR MAGNITUDES
IN USE TODAY

100 BC . GREEK ASTRONOMER POSIDONIUS
erroneously calculates earth's
circumference is 18,000 miles
(accepted into middle ages)

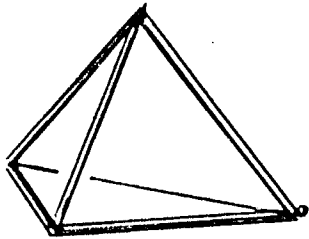
140 AD PTOLEMY - ALMAGEST
- INVENTS EPICYCLES

DR T J OSLER

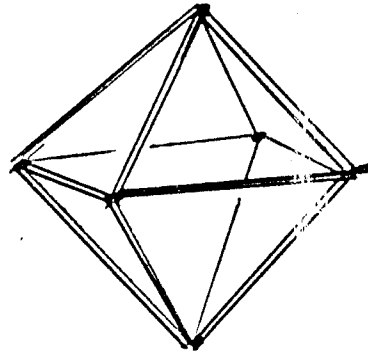
THE FIVE REGULAR (PLATONIC) SOLIDS

①
HM
OSLER
12

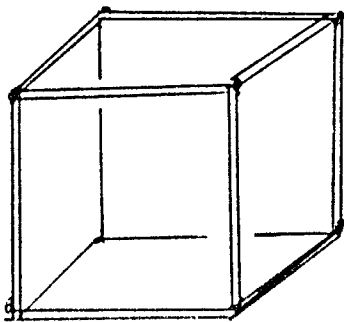
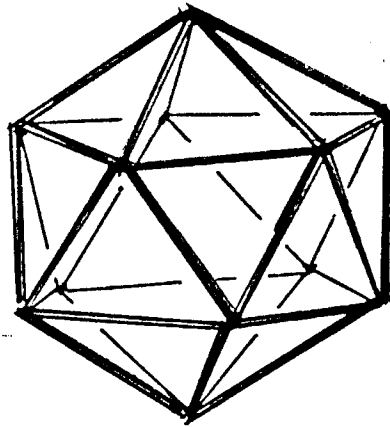
TETRAHEDRON



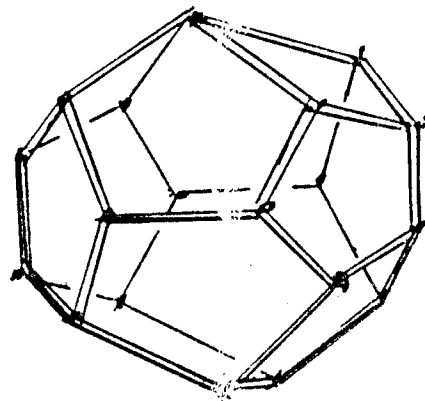
OCTAHEDRON



ICOSAHEDRON



CUBE



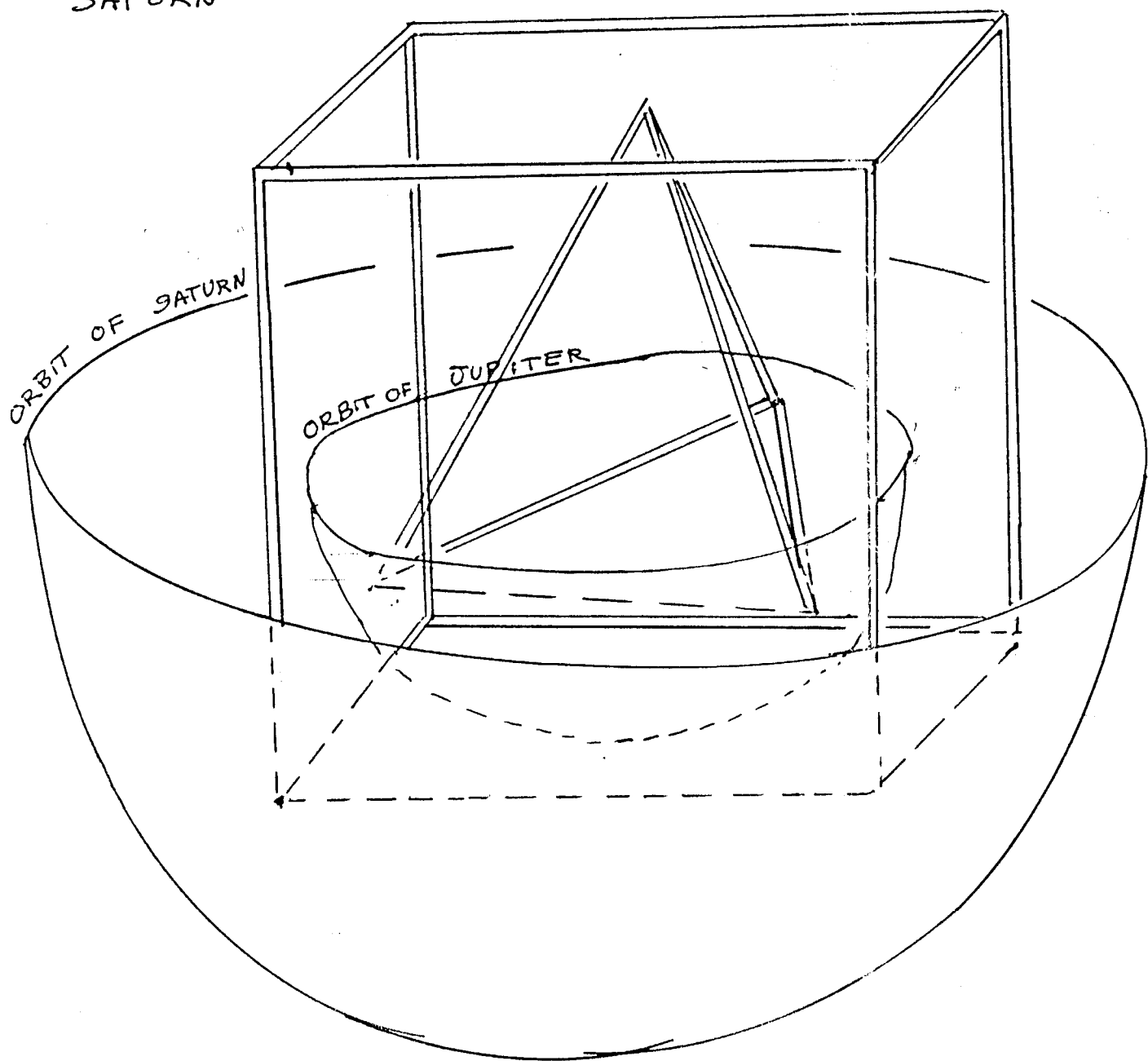
DODECAHEDRON

DR T J OSLER

KEPLER'S MODEL OF 5
REGULAR SOLIDS BETWEEN
THE PLANETARY SPHERES

②
HM
OSLER
13

MERCURY	OCTAHEDRON	— 8	FACES
VENUS	ICOSAHEDRON	— 20	"
EARTH	DODECAHEDRON	— 12	"
MARS	TETRAHEDRON	— 4	"
JUPITER	CUBE	— 6	"
SATURN			



DR T J OSLER

(3)
HM
OSLER
19

PLANETARY DATA TO TEST

KEPLER'S THIRD LAW

$$R^3 \propto T^2$$

PLANET	ORBITAL RADIUS IN MILLIONS OF MILES R	PLANET'S YEAR IN DAYS = T	R^3 (MILES) ³	T^2 (DAYS)	R^3/T^2 ($\frac{\text{MILES}^3}{\text{DAYS}^2}$)
MERCURY	35,96	87,97	$46,50 \times 10^{21}$	7739	$6,009 \times 10^{18}$
VENUS	67,20	224,7	$303,5 \times 10^{21}$	50490	$6,011 \times 10^{18}$
EARTH	92,90	365,3	$801,8 \times 10^{21}$	133400	$6,010 \times 10^{18}$
MARS	141,6	687,0	2839×10^{21}	472100	$6,015 \times 10^{18}$
JUPITER	483,3	4332	112900×10^{21}	18770000	$6,015 \times 10^{18}$
SATURN	886,1	10760	695700×10^{21}	115800000	$6,008 \times 10^{18}$


 HISTORY OF MATH
 DR T J OSLER

HOMEWORK ON SOLVING CUBIC EQUATIONS

- (1) $x^3 + 6x - 7 = 0$ ANS $1, \frac{1}{2}(-1 \pm 3i\sqrt{3})$
- (2) $x^3 - 18x - 35 = 0$ ANS $5, \frac{1}{2}(-5 \pm i\sqrt{3})$
- (3) $x^3 - 3x^2 + 4 = 0$ ANS $-1, 2, 2$
- (4) $3x^3 - 5x + 14 = 0$ ANS $-2, \frac{1}{3}(3 \pm 2i\sqrt{3})$
- (5) $x^3 + 3x^2 - 6x + 20 = 0$ ANS $-5; 1 \pm i\sqrt{3}$
- (6) $x^3 + 6x^2 - 3x + 104 = 0$ ANS $-8; 1 \pm 2i\sqrt{3}$
- (7) $x^3 - 3x^2 + 9x - 5 = 0$ ANS $1 + \sqrt[3]{2} - \sqrt[3]{4},$
 $\frac{1}{2}(2 + \sqrt[3]{4} - \sqrt[3]{2}) \pm \frac{1}{2}i\sqrt{3}(\sqrt[3]{4} + \sqrt[3]{2})$

HOMEWORK SOLVING QUARTIC EQUATIONS

- (1) $x^4 + x^2 + 14x - 6 = 0$ $-1 \pm \sqrt{2}, 1 \pm i\sqrt{5}$
- (2) $x^4 - 3x^2 + 6x - 2 = 0$ $-1 \pm \sqrt{2}, 1 \pm i$
- (3) $x^4 - 4x^2 + 8x + 35 = 0$ $-2 \pm i, 2 \pm i\sqrt{3}$
- (4) $x^4 - 4x^3 - 4x^2 - 4x - 5 = 0$ $-1, 5, \pm i$
- (5) $2x^4 + 8x^3 - 9x^2 + 4x - 5 = 0$ $-5, 1, \pm \frac{i\sqrt{2}}{2}$

FROM EUCLID'S ELEMENTS

①
16

BOOK I.

DEFINITIONS.

1. A **point** is that which has no part.
2. A **line** is breadthless length.
3. The extremities of a line are points.
4. A **straight line** is a line which lies evenly with the points on itself.
5. A **surface** is that which has length and breadth only.
6. The extremities of a surface are lines.
7. A **plane surface** is a surface which lies evenly with the straight lines on itself.
8. A **plane angle** is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.
9. And when the lines containing the angle are straight, the angle is called **rectilinear**.
10. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is **right**, and the straight line standing on the other is called a **perpendicular** to that on which it stands.
11. An **obtuse angle** is an angle greater than a right angle.
12. An **acute angle** is an angle less than a right angle.
13. A **boundary** is that which is an extremity of anything.
14. A **figure** is that which is contained by any boundary or boundaries.
15. A **circle** is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another ;

16. And the point is called the **centre** of the circle.

17. A **diameter** of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.

18. A **semicircle** is the figure contained by the diameter and the circumference cut off by it. And the centre of the semicircle is the same as that of the circle.

19. **Rectilinear figures** are those which are contained by straight lines, **trilateral** figures being those contained by three, **quadrilateral** those contained by four, and **multi-lateral** those contained by more than four straight lines.

20. Of trilateral figures, an **equilateral triangle** is that which has its three sides equal, an **isosceles triangle** that which has two of its sides alone equal, and a **scalene triangle** that which has its three sides unequal.

21. Further, of trilateral figures, a **right-angled triangle** is that which has a right angle, an **obtuse-angled triangle** that which has an obtuse angle, and an **acute-angled triangle** that which has its three angles acute.

22. Of quadrilateral figures, a **square** is that which is both equilateral and right-angled; an **oblong** that which is right-angled but not equilateral; a **rhombus** that which is equilateral but not right-angled; and a **rhomboid** that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called **trapezia**.

23. **Parallel** straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

POSTULATES.

Let the following be postulated :

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any centre and distance.
4. That all right angles are equal to one another.

2
17

BOOK I. PROPOSITIONS.

PROPOSITION I.

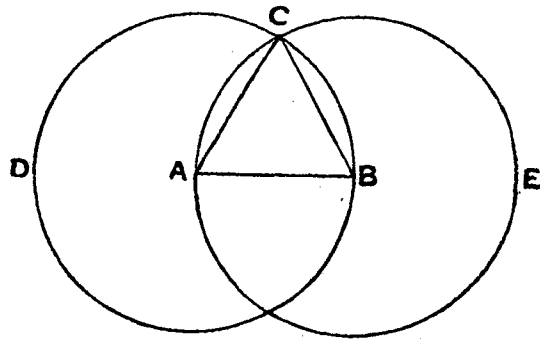
On a given finite straight line to construct an equilateral triangle.

Let AB be the given finite straight line.

Thus it is required to construct an equilateral triangle on the straight line AB .

With centre A and distance AB let the circle BCD be described ; [Post. 3]

again, with centre B and distance BA let the circle ACE be described ; [Post. 3]



and from the point C , in which the circles cut one another, to the points A, B let the straight lines CA, CB be joined.

[Post. 1]

Now, since the point A is the centre of the circle CDB , AC is equal to AB .

[Def. 15]

Again, since the point B is the centre of the circle CAE , BC is equal to BA .

[Def. 15]

But CA was also proved equal to AB ;

therefore each of the straight lines CA, CB is equal to AB .

And things which are equal to the same thing are also equal to one another ;

[C. N. 1]

therefore CA is also equal to CB .

Therefore the three straight lines CA, AB, BC are equal to one another.

5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

COMMON NOTIONS.

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
- [7] 4. Things which coincide with one another are equal to one another.
- [8] 5. The whole is greater than the part.

DEFINITION I.

Σημείον ἐστίν, οὐ μέρος οὐθέν.

A point is that which has no part.

An exactly parallel use of μέρος (ἐστίν) in the singular is found in Aristotle, *Metaph.* 1035 b 32 μέρος μὲν οὖν ἐστὶ καὶ τοῦ εἶδους, literally "There is a part even of the form"; Bonitz translates as if the plural were used, "Theile giebt es," and the meaning is simply "even the form is *divisible* (into parts)." Accordingly it would be quite justifiable to translate in this case "A point is that which is *indivisible into parts*."

Martianus Capella (5th c. A.D.) alone or almost alone translated differently, "Punctum est cuius pars nihil est," "a point is that a part of which is *nothing*." Notwithstanding that Max Simon (*Euclid und die sechs planimetrischen Bücher*, 1901) has adopted this translation (on grounds which I shall presently mention), I cannot think that it gives any sense. If a part of a point is *nothing*, Euclid might as well have said that a point is *itself* "nothing," which of course he does not do.

Pre-Euclidean definitions.

It would appear that this was not the definition given in earlier textbooks; for Aristotle (*Topics* VI. 4, 141 b 20), in speaking of "the definitions" of point, line, and surface, says that they *all* define the prior by means of the posterior, a point as an extremity of a line, a line of a surface, and a surface of a solid.

The first definition of a point of which we hear is that given by the Pythagoreans (cf. Proclus, p. 95, 21), who defined it as a "monad having position" or "with position added" (μονὰς προσλαβοῦσα θέσιν). It is frequently used by Aristotle, either in this exact form (cf. *De anima* I. 4, 409 a 6) or its equivalent: e.g. in *Metaph.* 1016 b 24 he says that that which is indivisible every way in respect of magnitude and *quâ* magnitude but has not position is a *monad*, while that which is similarly indivisible and has position is a *point*.

Plato appears to have objected to this definition. Aristotle says (*Metaph.*

4
19

Therefore the triangle ABC is equilateral; and it has been constructed on the given finite straight line AB .

(Being) what it was required to do.

1. On a given finite straight line. The Greek usage differs from ours in that the definite article is employed in such a phrase as this where we have the indefinite. ἐπι τῆς δοθείσης εὐθείας πεπερασμένης, "on the given finite straight line," i.e. the finite straight line which we choose to take.

3. Let AB be the given finite straight line. To be strictly literal we should have to translate in the reverse order "let the given finite straight line be the (straight line) AB "; but this order is inconvenient in other cases where there is more than one datum, e.g. in the setting-out of I. 2, "let the given point be A , and the given straight line BC ," the awkwardness arising from the omission of the verb in the second clause. Hence I have, for clearness' sake, adopted the other order throughout the book.

8. let the circle BCD be described. Two things are here to be noted, (1) the elegant and practically universal use of the perfect passive imperative in constructions, γεγράφθω meaning of course "let it have been described" or "suppose it described," (2) the impossibility of expressing shortly in a translation the force of the words in their original order. κύκλος γεγράφθω ὁ $BΓΔ$ means literally "let a circle have been described, the (circle, namely, which I denote by) BCD ." Similarly we have lower down "let straight lines, (namely) the (straight lines) CA, CB , be joined," ἐπεζεύχθωσαν εὐθεῖαι αἱ $ΓΑ, ΓΒ$. There seems to be no practicable alternative, in English, but to translate as I have done in the text.

13. from the point C ... Euclid is careful to adhere to the phraseology of Postulate 1 except that he speaks of "joining" (ἐπεζεύχθωσαν) instead of "drawing" (γράφειν). He does not allow himself to use the shortened expression "let the straight line FC be joined" (without mention of the points F, C) until I. 5.

20. each of the straight lines CA, CB , ἑκάτερα τῶν $ΓΑ, ΓΒ$ and 24. the three straight lines CA, AB, BC , αἱ τρεῖς αἱ $ΓΑ, ΑΒ, ΒΓ$. I have, here and in all similar expressions, inserted the words "straight lines" which are not in the Greek. The possession of the inflected definite article enables the Greek to omit the words, but this is not possible in English, and it would scarcely be English to write "each of CA, CB " or "the three CA, AB, BC ."

It is a commonplace that Euclid has no right to assume, without promising some postulate, that the two circles will meet in a point C . To supply what is wanted we must invoke the Principle of Continuity (see note thereon above, p. 235). It is sufficient for the purpose of this proposition and of I. 22, where there is a similar tacit assumption, to use the form of postulate suggested by Killing. "If a line [in this case e.g. the circumference ACE] belongs entirely to a figure [in this case a plane] which is divided into two parts [namely the part enclosed within the circumference of the circle BCD and the part outside that circle], and if the line has at least one point common with each part, it must also meet the boundary between the parts [i.e. the circumference ACE must meet the circumference BCD]."

Zeno's remark that the problem is not solved unless it is taken for granted that two straight lines cannot have a common segment has already been mentioned (note on Post. 2, p. 196). Thus, if AC, BC meet at F before reaching C , and have the part FC common, the triangle obtained, namely FAB , will not be equilateral, but FA, FB will each be less than AB . But Post. 2 has already laid it down that two straight lines cannot have a common segment.

Proclus devotes considerable space to this part of Zeno's criticism, but satisfies himself with the bare mention of the other part, to the effect that it is also necessary to assume that two circumferences (with different centres) cannot have a common part. That is, for anything we know, there may be any number of points C common to the two circumferences ACE, BCD . It is not until III. 10 that it is proved that two circles cannot intersect in more

5
20