Part 1

Outline for the

History of Mathematics

Dr Osler

THE HISTORY OF MATHEMATICS
AN INTRODUCTION

DAVID M BURTON (ALLYN & BACON 1985)

CHAP 1 EARLY NUMBER SYSTEMS & SYMBOLS

PRIMITIVE COUNTING

EVOLUTION OF COUNTING LOST

TALLY

- from French tailler "to cut"
- "TALLY STICKS"
- WOODEN TALLY STICKS USED IN SWITZERLAND EARLY THIS CENTURY

NUMBER RECORDING OF EGYPTIANS & GREEKS

AS EARLY AS 3500 BC EGYPTIANS HAD FULL NUMBER SYSTEM

HAD SYMBOLS

𓏺	1
𓎆	10
𓍢	100
𓆼	1000

etc

𓏺𓏺 𓎆𓎆𓎆 𓍢𓍢 𓆼 MEANS 1232

AROUND 500 BC GREEKS USED SYSTEM WITH 27 SYMBOLS (alphabet + 3 Phoenician letters)

$\psi\ \pi\ \delta = 784$ not a positional system

700 80 4

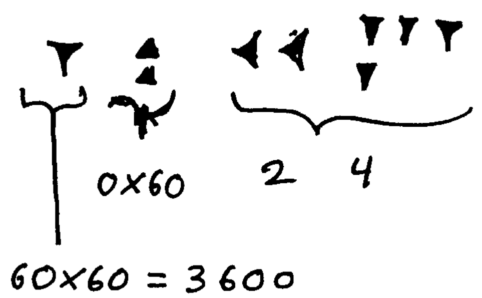
NUMBER RECORDING OF BABYLONIANS

USED POSITIONAL NUMBER SYSTEM

- a few symbols used to write all numbers
- SEXAGESIMAL
  - BASE 60

- ONE (USED UP TO NINE TIMES)
- TEN (" " " FIVE ")

At first there was no zero.
Later (300 BC) was used for an empty place, but never in first position.



= 3624

Babylonians never developed a full positional system.

Chap 2 Math in Early Civilizations

The Rhind Papyrus

Date 1650 BC

Egyptian Arithmetic

Multiplication

- successive doubling

EX 19×71

✓	1	71
✓	2	142
	4	284
	8	568
✓	16	1136
	19	1349 ← Add checked items

Division

- divisor doubled to give dividend

EX $91 \div 7$

✓	1	7
	2	14
✓	4	28
✓	8	56
	13	91

FRACTIONS

- CONSIDERED ONLY THOSE OF FORM $\frac{1}{n}$

$$\frac{6}{7} = \frac{1}{2} + \frac{1}{4} + \frac{1}{14} + \frac{1}{28}$$

THREE PROBLEMS FROM RHIND PAPYRUS

PROB 24 – a quantity and it's $\frac{1}{7}$ added become 19. What is the quantity?

$$x + \frac{x}{7} = 19 \quad \text{or} \quad \frac{8x}{7} = 19$$

PROB 28 – Think of a number, add $\frac{2}{3}$ of this number to itself. ~~Now subtract~~ ! From this sum subtract $\frac{1}{3}$ its value, & say what the answer is. Suppose the answer is 10. Then take away $\frac{1}{10}$ of this 10 giving 9. This was the original number.

This is the Identity!

$$\left[\left(n + \frac{2n}{3}\right) - \frac{1}{3}\left(n + \frac{2n}{3}\right)\right] - \frac{1}{10}\left[\left(n + \frac{2}{3}n\right) - \frac{1}{3}\left(n + \frac{2n}{3}\right)\right] = n$$

PROB 79

HOUSES	7	$= 7$
CATS	49	$= 7^2$
MICE	343	$= 7^3$
SHEAVES	2401	$= 7^4$
HEKATS	16807	$= 7^5$
TOTAL	19 607	

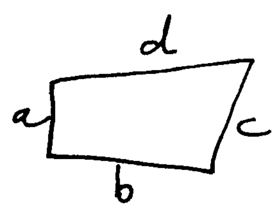
Did the Egyptians know

$$a + ar + ar^2 + \dots + ar^{n-1} = a\,\frac{r^n - 1}{r-1}$$

$$7 + 7\cdot 7 + 7\cdot 7^2 + \dots + 7\cdot 7^4 = 7\cdot \frac{7^5 - 1}{7-1}$$

We don't know.

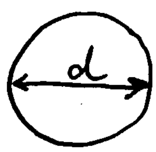
EGYPTIAN GEOMETRY



$$A = \tfrac{1}{4}(a+c)(b+d)$$

incorrect but close for a rectangle

"Empirical formula"



$$A = \left(d - \frac{d}{9}\right)^2 = \left(\frac{8d}{9}\right)^2$$

Compared to $\pi\left(\frac{d}{2}\right)^2$ we get

$$\pi = 4\left(\tfrac{8}{9}\right)^2 = 3.1605$$

Babylonian Mathematics

- had Pythagorean Thm
- sexagesimal no. sys, led to a highly developed algebra
- indefatigable compilers of tables
  squares, cubes, $\sqrt{\ }$, $\sqrt[3]{\ }$
  $n^2 + n^3$ (to solve $x^3 + x^2 = a$)
- also

4	15
5	12
6	10
7	
8	7;30
9	6;40
10	6
x	y
⋮	⋮

← does not appear since $\frac{1}{7}$ is a repeating dec.

→ where $xy = 60$

so that $\frac{1}{x} = \frac{y}{60}$

$\frac{1}{4} = 0;15$

$\frac{1}{5} = 0;12$

etc

- could solve $x^2 + ax = b$ using

$$x = \sqrt{\left(\frac{a}{2}\right)^2 + b} - \frac{a}{2}$$

PLIMPTON 322

— 1900 BC TO 1600 BC

— Proves Pythagorean Thm was known to Babylonians over 1000 yrs before Pythagorous was born

— The table contains integral solutions of

$$x^2 + y^2 = z^2$$

x	z
119	169
3367	4825 (11521)
4601	6649
⋮	⋮

$$(169)^2 - (119)^2 = (120)^2$$

$$z^2 - x^2 = \text{perfect square}$$

How can these be obtained?

(1) $\left(\frac{z}{x}\right)^2 - \left(\frac{y}{x}\right)^2 = 1$

(2) $\alpha = \frac{z}{x}, \quad \beta = \frac{y}{x}$

$$\alpha^2 - \beta^2 = 1$$

(3) $(\alpha+\beta)(\alpha-\beta) = 1$

(4) $\alpha+\beta = \frac{m}{n}, \quad \alpha - \beta = \frac{n}{m}$

where m & n are integers

$$\alpha = \frac{1}{2}\left(\frac{m}{n} + \frac{n}{m}\right), \quad \beta = \frac{1}{2}\left(\frac{m}{n} - \frac{n}{m}\right)$$

$$\alpha = \frac{m^2+n^2}{2mn}, \quad \beta = \frac{m^2-n^2}{2mn}$$

We don't know if this was the method used by Babylonian mathematicians

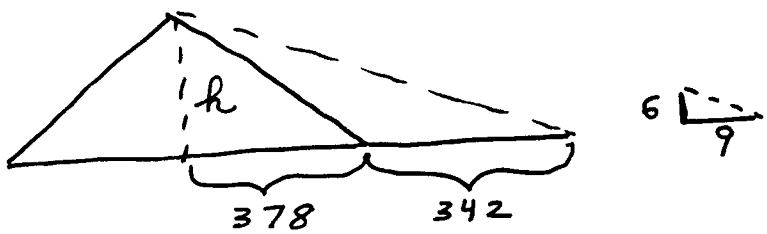
Take $x = 2mn$, $y = m^2 - n^2$, $z = m^2 + n^2$

$m = 12$, $n = 5$ gives first entry in the table

CHAP 3 BEGINNINGS OF GREEK MATH

THALES OF MILETUS (625-547 BC)

- 1st known Greek math,
- developed thms via rigorous proof
  - 1st known to do so
- Measured Height of great pyramid using length of shadow of his staff



(all in ft)

$$\frac{h}{378+342} = \frac{6}{9}$$

$$h = \frac{2}{3}(720) = 480 \text{ ft}$$

- left no written record
- may have been teacher of PYTHAGORAS

PYTHAGORAS OF SAMOS (569-501 BC)

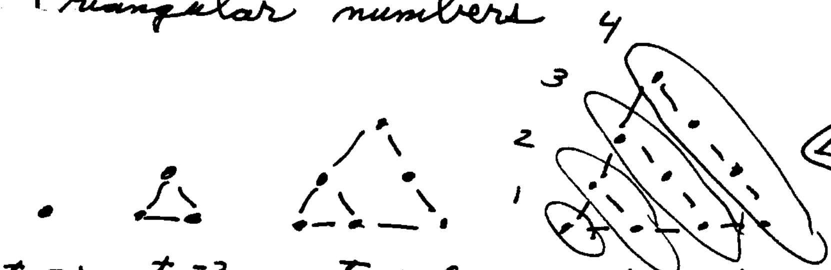
- started school at Croton (South Italy)
- ruling principles in universe were math,
  - "Everything is number"
- music was best example
  - pluck 2 strings whose lengths form simple fraction to get a harmonious sound

- invented idea of "music of the spheres" (which only Pythagoras could hear)

- work of Pythagoreans comes to us through Nicomachus (100 ad)

FIGURATIVE NUMBERS

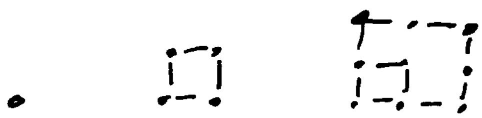
Triangular numbers



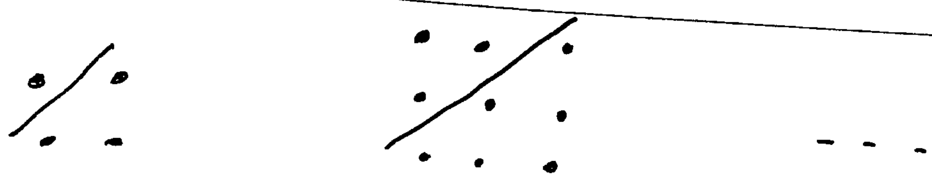
$t_1 = 1$ $t_2 = 3$ $t_3 = 6$ $t_4 = 10$

~~FIGURATIVE NUMBERS~~

SQUARE NUMBERS



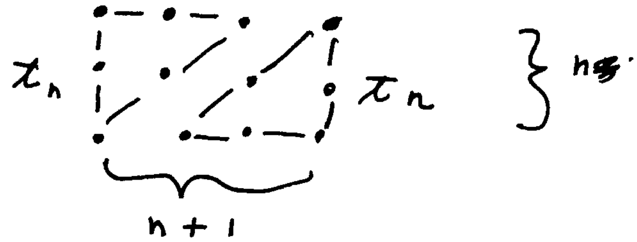
$S_1 = 1$ $S_2 = 4$ $S_3 = 9$ $S_4 = 16$



$S_2 = t_1 + t_2$ $S_3 = t_2 + t_3$

$$t_n = t_{n-1} + n$$

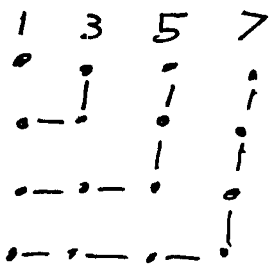
$$t_n = 1 + 2 + 3 + \cdots + n$$ as seen from figure



$$2t_n = n(n+1)$$

$$t_n = \frac{n(n+1)}{2} = 1+2+3+\cdots+n$$

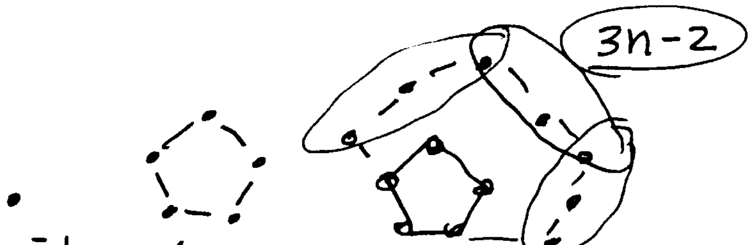
Also show same sum as $(n+1)$ times $\frac{n}{2}$



$$1+3+5+7 = 4^2$$

$$1+3+5+\cdots+(2n-1) = n^2$$

PENTAGONAL NUMBERS

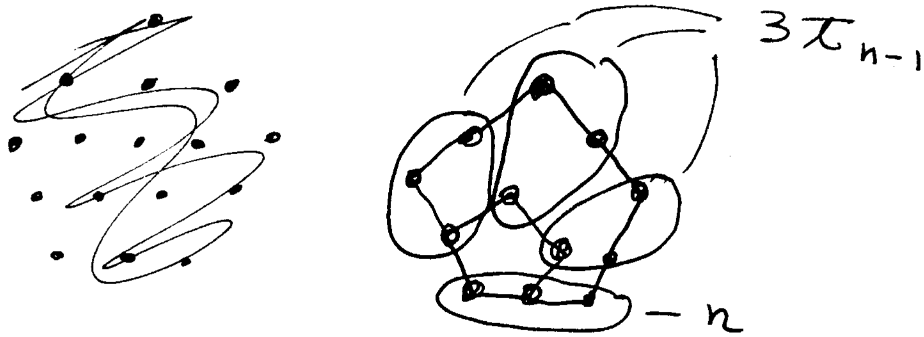


$p_1 = 1$ $p_2 = 5$ $p_3 = 12$ $p_4 = 22$

$$p_n = 1+4+7+\cdots+(3n-2) = (3n-1)\frac{n}{2}$$

Another derivation

$$\underbrace{p_n = n + 3t_{n-1}}_{\text{from figure below}} = n + 3\underbrace{\tfrac{n}{2}(n-1)}_{\text{previous value}} = \frac{n(3n-1)}{2}$$



see page 105 bottom

In 1665 Pascal wrote "Treatise on Fig. Nos."

- he conjectured

"every pos. int. is the sum of at most 3 triangular nos"

$16 = 6 + 10$

$25 = 1 + 3 + 21$

$39 = 3 + 15 + 21$

1st Proved by Gauss in 1801

To find Sum $\boxed{\displaystyle\sum_{1}^{n} k^3}$

$$\underbrace{1 + 3 + 5 + \cdots + (2k-1)}_{k \text{ terms}} = k^2$$

$$(1) \quad [k(k-1)+1] + [k(k-1)+3] + \cdots + [k(k-1) + (2k-1)]$$

$$= k[k(k-1)] + k^2 = k^3$$

Take $k = 1, 2, 3, \cdots$ in (1) to get

$$\begin{aligned}
&1 && = 1^3 \\
&3 + 5 && = 2^3 \\
&7 + 9 + 11 && = 3^3 \\
&13 + 15 + 17 + 19 && = 4^3 \\
&\vdots && \\
&\sum_{k=1}^{n} [k(k-1) + (2k-1)] && = n^3
\end{aligned}$$

→ add these up to get

$$1 + 3 + 5 + 7 + \cdots + \underbrace{\begin{bmatrix} n(n-1) + \\ (2n-1) \end{bmatrix}}_{} = 1^3 + 2^3 + 3^3 + \cdots + n^3$$

$$n^2 + n - 1 = 2\left[\frac{n(n+1)}{2}\right] - 1$$

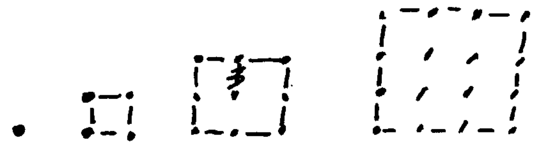
Thus there are $\frac{n(n+1)}{2}$ terms on L.H.S.

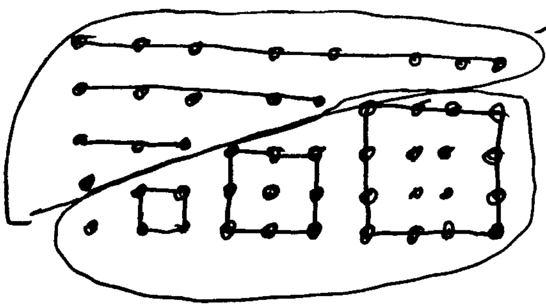
This sum is $\left[\frac{n(n+1)}{2}\right]^2 = t_n^2$

Thus $\displaystyle\sum_{1}^{n} k^3 = t_n^2$

START 3

$$\boxed{\text{To Find Sum } \sum_1^n k^2}$$





$$(1^2+2^2+3^2+4^2)+(1+3+6+10)$$
$$=(1+2+3+4)(4+1)$$

or in general

$$(1^2+2^2+3^2+\cdots+n^2)+(t_1+t_2+t_3+\cdots+t_n)=t_n(n+$$

$$\sum_1^n k^2 + \left(\frac{1\cdot2}{2}+\frac{2\cdot3}{2}+\frac{3\cdot4}{2}+\cdots+\frac{n(n+1)}{2}\right)=\frac{n(n+1)}{2}$$

$\downarrow$

$$S+\tfrac{1}{2}\Big(1(1+1)+2(2+1)+3(3+1)+\cdots+n(n+1)\Big)=\frac{n(n+}{2}$$

$$S+\tfrac{1}{2}\Big(1^2+2^2+3^2+\cdots+n^2+1+2+3+\cdots+n\Big)= \;''$$

$$S+\tfrac{1}{2}\Big(\quad S \quad + \frac{n(n+1)}{2}\Big) = \;''$$

$$\frac{3}{2}S+\frac{n(n+1)}{4}=\frac{n(n+1)^2}{2}$$

$$S=\frac{2}{3}\left[\frac{n(n+1)^2}{2}-\frac{n(n+1)}{4}\right]=\frac{n(n+1)}{3}\left[n+1-\frac{1}{2}\right]$$
$$=\frac{n(n+1)(2n+1)}{6}$$

See Monthly Vol 93, # 6, June-July 1986

A. W. F. EDWARDS

"A QUICK ROUTE TO SUMS OF POWERS"

pp. 451-455

This shows $\sum_{1}^{n} k^{p}$, $p = 1, 2, 3, \ldots$

At this point I discuss how modern math research is done.

- Prof. Societies
  - AMS
  - SIAM
  - Nat Acad Sci
- UNIVERSITY vs COLLEGE
  - research — teach

Prof does paper.

↓

JOURNAL → EDITOR → REFEREE 1

EDITOR → REF. 2

EDITOR → JOURNAL

- Take 1.5 yrs.

Show handout for referees on next page.

13-2

INFORMATION FOR REFEREES

OF ARTICLES SUBMITTED TO AMERICAN MATHEMATICAL SOCIETY FOR PUBLICATION

CRITERIA FOR ACCEPTING A PAPER

1. It must be correct. While this is basically the author's responsibility, the referee certainly must be reasonably convinced.

2. It must be new in a nontrivial sense; e.g., a collection of new definitions and deductions therefrom is not publishable unless the author shows that (or unless it is clear that) it simplifies or solves some existing problems of reasonable importance. Similarly, a new theorem with an old proof may not be publishable.

3. It must be of interest to an appreciable number of readers, not just the author, the author's students, and one or two colleagues.

4. It must be clearly written; at least the referee should be able to understand it without undue pain. In a paper which is otherwise marginal, failure in this respect could be cause for rejection.

ITEMS TO BE CHECKED

1. Title should be informative.

2. 1980 Mathematics Subject Classification categories should identify the fields of an article accurately.

3. Abstract should summarize the results of the paper. The main purpose of the abstract is to enable readers to take in the nature and results of the paper quickly and to decide whether they wish to read the entire paper. ZENTRALBLATT now publishes author abstracts instead of reviews so the abstract may also appear there.

4. If key words or index terms are included (they are optional), they should indicate important topics considered.

(Referees should be familiar with the contents of the Instructions for Authors of Papers for AMS Journals. The four items listed above are described in detail in the Instructions.)

5. Items listed in the bibliography should be relevant to the subject of the paper. Referees need not be concerned about the style of the bibliography.

REFEREE'S REPORT. The referee is asked to recommend acceptance, rejection, or revision of each paper. Each recommendation should be clear and well founded and should be based on a thorough reading of the paper. Since the referee's report will be transmitted (anonymously) to the author, reasons for a recommendation should be carefully stated, preferably on a plain piece of paper without identifying water marks so that the report can be copied without retyping and mailed to the author without revealing the identity of the referee.

REFEREEING TIME. Papers of less than ten manuscript pages should not take more than a month to referee. Longer papers may require more time, a manuscript of 50 pages could take three months; no paper should require more than four months.

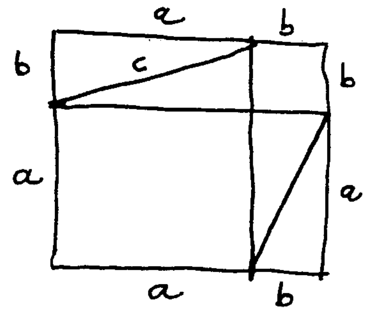
mdb 5/27/80

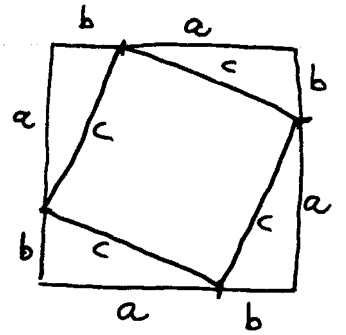
HOME WORK PROBLEMS

P. 109 1, 7, 12, 13, 16

THE PYTHAGOREAN PROBLEM

Possible Proof





AREA =

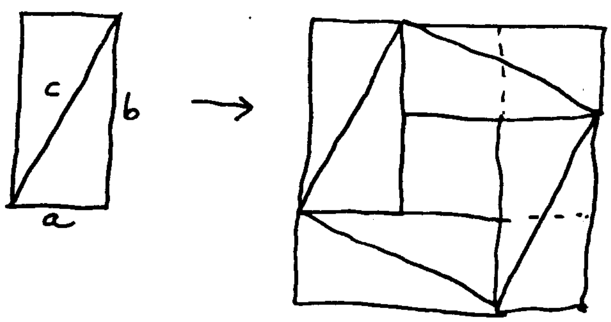
$a^2 + b^2 + 2ab$

AREA =

$c^2 + 4\left(\frac{1}{2}ab\right)$

$$a^2 + b^2 = c^2$$

Another Proof – China 600 BC



$$\text{Area} = c^2 + 4\left(\frac{ab}{2}\right)$$

$$= a^2 + b^2 + 2ab$$

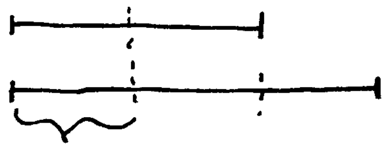
Thus $c^2 = a^2 + b^2$

from "Arithmetic Classic of the Gnomon and the Circular Paths of Heaven"

- Most important achievement of Pythagorean school in its influence on the number concept was the discovery of the "irrational"

- Intuitively they felt that every two line segments had a common measure,

ex

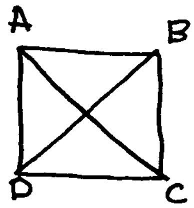


$\frac{2}{3}$

segment fits in both lengths a whole number of times

START 4

- Oldest known proof that $\sqrt{2}$ is irrational is in Euclid's Elements - book ten



Assume δ is common measure of AC and AB

$AC = m\delta$, $AB = n\delta$

m & n integers

$$\frac{AC}{AB} = \frac{m}{n}$$

Assume common factors of m & n cancelled

$$\frac{(AC)^2}{(AB)^2} = \frac{m^2}{n^2}$$

But $(AB)^2 + (AB)^2 = (AC)^2$

$$2(AB)^2 = (AC)^2$$

$$2 = \frac{(AC)^2}{(AB)^2}$$

Thus $\frac{m^2}{n^2} = 2$, $m^2 = 2n^2$, m is even

Thus $m = 2k$

$$m^2 = 2n^2$$
$$(2k)^2 = 2n^2$$
$$4k^2 = 2n^2$$
$$2k^2 = n^2$$

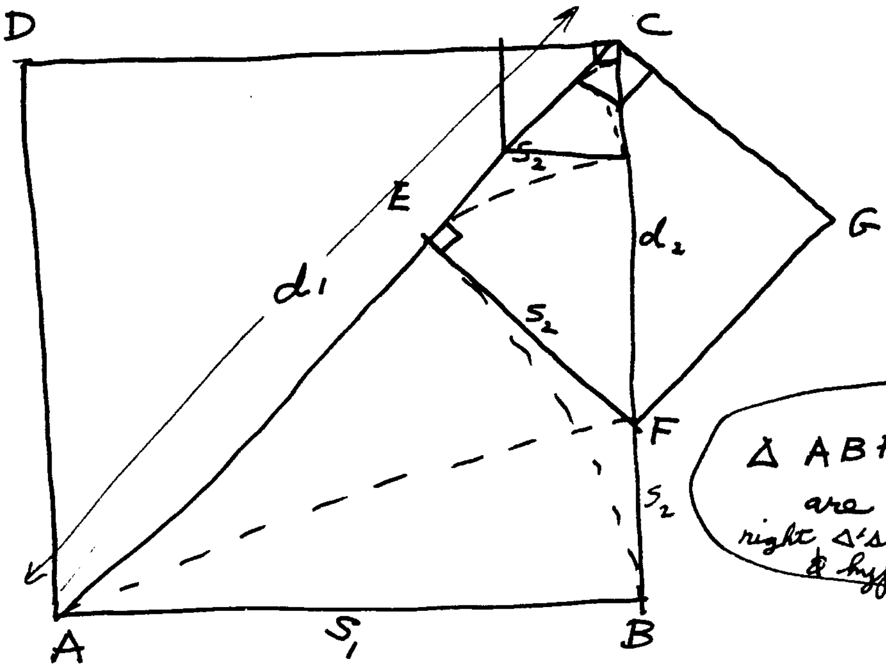
Thus n is even $\leftarrow$ contradiction since both m and n can't be even. (common factors were cancelled)

Babylonians had approximated $\sqrt{2}$ as

$$\sqrt{2} \cong 1;\ 24,\ 51,\ 10 = 1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3}$$
$$\cong 1.414213$$

This close to our value $\sqrt{2} = 1.414213562\ldots$

Older proof that diagonal & side of a square are incommensurable.



Δ ABF & ΔAEF are congruent right Δ's with same leg & hypotenus

$$s_2 = d_1 - s_1, \qquad d_2 = s_1 - \overline{FB}$$
$$d_2 = s_1 - s_2$$

$$s_n = d_{n-1} - s_{n-1}, \qquad d_n = s_{n-1} - s_n$$

Assume ~~Suppose~~ $s_1 = M_1\delta$, $d_1 = N_1\delta$

Where δ is the common measure.

$$s_2 = d_1 - s_1 = N_1\delta - M_1\delta = (N_1 - M_1)\delta = M_2\delta$$
$$d_2 = s_1 - s_2 = M_1\delta - M_2\delta = (M_1 - M_2)\delta = N_2\delta$$

$M_1 > M_2 > M_3 > \cdots \geq 1$

$N_1 > N_2 > N_3 > \cdots \geq 1$

these are impossible since there are only a finite no. of integers between M_1 and 1.

- irrationals challenged the philosophy that number is the essence of all things.

- EUDOXUS OF CNIDOS (408 – 355 BC) resolved the problem by ignoring the idea of number and using only geometric lengths.
  - Arithmetic theory of numbers is renounced.
  - Geometry is the more general science and is the basis of rigorous math for the next 2000 years.

HOMEWORK
Page 125; 8 and 17

THREE CONSTRUCTION PROBLEMS OF ANTIQUITY

(1) Squaring the circle
- Find a square whose area equals that of a given circle.

(2) Duplicating the cube.
- Find the side of a cube having double the volume of a given cube

(3) Trisecting the angle

(429 - 348 BC)

- Plato required "RULER & COMPASS ONLY"

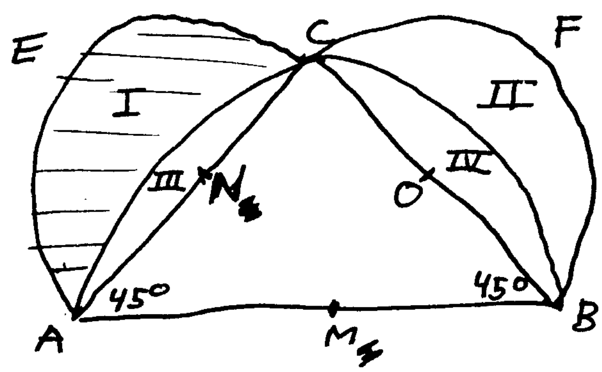
  1. With ruler a line can be drawn between two given points
  2. With compass, a circle with a given center and radius can be drawn.

- Not until the 19th century were these constructions proved to be impossible.

- Hippocrates of Chios (460 - 380 BC)
  (not father of medicine)

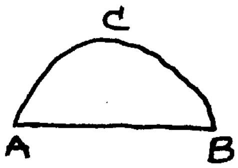
  Showed that certain plane regions with curved boundaries are squarable.

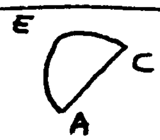
  - Consider the lune I



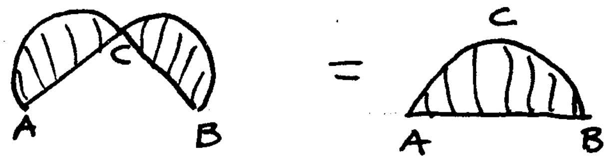
Arc ACB is a semi circle with center M
" AEC " " " N
" CFB O

$$\frac{\text{Area}}{\text{Area}} = \frac{\overline{AB}^{2}}{\overline{AC}^{2}} = 2$$

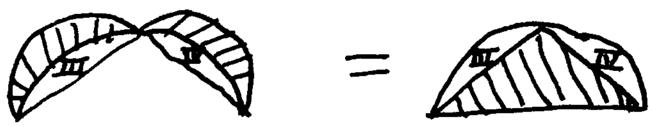




THUS



Now subtract area in regions III and IV



THUS $\quad 2\,(\text{Area of lune I}) = \frac{1}{2}(AC)^2$

or

$$(\text{Area of lune I}) = \left(\frac{AC}{2}\right)^2$$

And lune I has been squared!

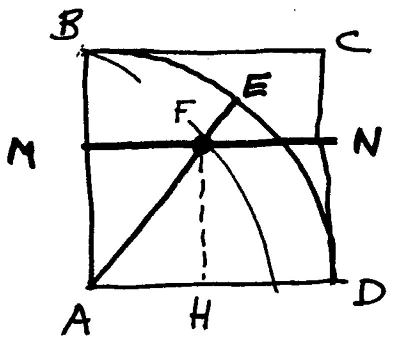
- For 2000 yrs no one could trisect the angle in 1837 Pierre Wantzel (1814–1848) gave the first rigorous proof that it's impossible.

THE QUADRATRIX OF HIPPIAS

Hippias of Elis (460 BC–)
invented a curve to trisect angles.

- he was a sophist
  - taught to argue well both sides of every argument
  - Plato made fun of him in one of his dialogues

QUADRATRIX



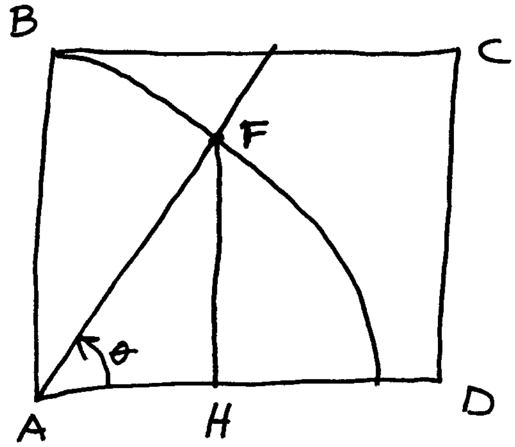
radius AE moves from AB to AD with constant velocity in time T

horizontal line MN moves from BC to AD with constant velocity in time T

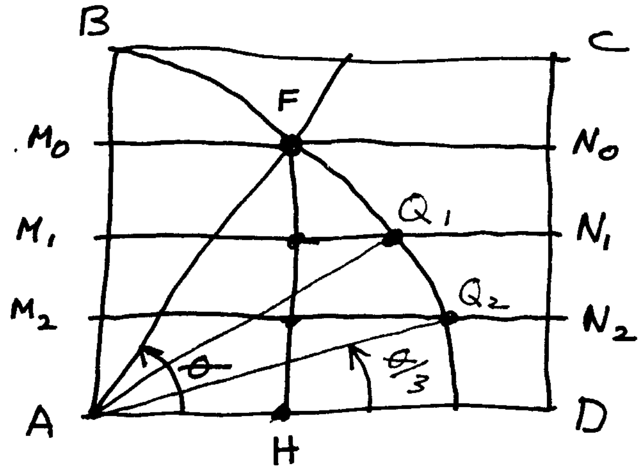
Locus of their intersections F is the desired curve.

START 5

GIVEN THE ANGLE θ TO TRISECT, LAY IT OUT ON THE QUADRATRIX AS $\angle DAF$



NEXT TRISECT $\overline{FH}$ AND DRAW HORRIZONALS THRU THESE POINTS $\overline{M_0 N_0}$, $\overline{M_1 N_1}$, $\overline{M_2 N_2}$



NEXT DRAW THE RADII $\overline{AQ_1}$ & $\overline{AQ_2}$ SHOWN

THE TIME FOR THE HORRIZONTAL $\overline{M_0 N_0}$ to drop to $\overline{M_1 N_1}$ = time to go on to $\overline{M_2 N_2}$ = time to reach bottom $\overline{AD}$, Also the time for radius $\overline{AF}$ to swing to $\overline{AQ_1}$ = time for it to go on to $\overline{AQ_2}$ = time to finish at $\overline{AD}$, SINCE THE RADIUS MOVIES AT CONSTANT ANGULAR VELOCITY, THE ANGLES

$\angle FAQ_1 = \angle Q_1 A Q_2 = \angle Q_2 A D$.

THUS θ HAS BEEN trisected

ABOUT 387 BC Plato founded an academy in the suburbs of Athens that would last 900 years

- intellectual center of Greece
- closed in 529 ad by Christian Emperor Justinian's orders
- over it's gates was a sign

"Let no man ignorant of geometry enter here"

- Plato gave math a favored place in the curriculum
- it is not known if he made any original contributions

- About 300 BC Ptolemy I founded a rival academy the "MUSEUM" in Alexandria
  - many math. went there

HOMEWORK

page 148; 2, 6, 7

Chapter 4

The First Alexandrian School: Euclid

Euclid & The Elements

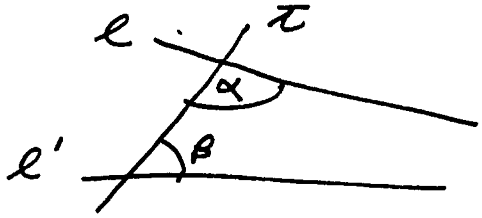
- End of 4^{th} cent. BC Math shifts from Greece to Egypt
- "Museum" is built at Alexandria
  - forerunner of modern university
  - fellows of Museum lived at King's expense
- In all history only span from 1600 - 1850 (Kepler to Gauss) compares with productivity of 300 - 100 BC.
- Great Alexandrian Library is established almost simultaneously with "Museum"
- A rival library was at Pergamon in western Asia Minor

- Euclid
  - unified collection of isolated discoveries
    - single deductive system
    - initial postulates
    - " definitions
    - " axioms
  - only the Bible has been more important in Western Thought & education than the Elements of Euclid
  - Euclid was author of at least ten other works

EUCLIDEAN GEOMETRY

~~the~~ Euclid's greatness
- not in original contributions
- in organizing vast body of facts
- introduced 5 postulates (or axioms)
- 23 definitions

Postulate 5 (parallel postulate)



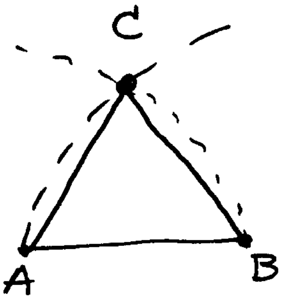
If $\alpha + \beta < 180°$ then l & l' intersect on this side.

- this is one of the most famous and controversial statements in math history
  - it was thought that Post. 5 should be provable
  - Mathematicians tried to derive it from the first 4 into the 19th century
    - these efforts led to discovery of "Non-Euclidean Geo."
  - Numerous flaws have been found in the Elements
    - no undefined terms
      - tried to define all technical vocabulary
    - axioms are woefully inadequate

Proposition 1

Given $\overline{AB}$, there is an equilateral triangle with $\overline{AB}$ as one side.

Proof:



Draw circles of radius $\overline{AB}$ intersecting at C

"How do we know that these circles will intersect?" This is not in the axioms!

David Hilbert (1862–1943)

- German
- Grundlagen der Geometrie
  Foundations of Geometry
  - most influential treatise on geo. in modern times
  - Euclidean geo
    - 6 undefined terms
    - 21 postulates

Supplement on "undefined terms"
-from "Patterns in the Sand"
by Bosstick & Cable - 1971 - Glencoe Press

- Sample Axiomatic System

> 1. There are at least two abas
> 2. For every two abas there is one and only one daba containing them.
> 3. Every daba contains at least one aba.
> 4. For every daba there is an aba not on that daba.

- here aba & daba are undefined terms
  - you can replace
    aba by airplane or point
    daba by runway or line

(airplane)

<u>Theorem</u> Every aba is on at least two dabas. (runways)

<u>Proof</u>

1. Let A denote an aba.
2. There is another aba, call it B - Axiom 1
3. There is a daba containing A & B - Axiom 2
4. There is another aba, call it C. - Axiom 4
5. There is another daba containing A & C - Axiom 2
6. Thus A is on two dabas. - END

<u>Homework</u>

Prove that there are at least 3 abas.

START CLASS #6

See p 580 at bottom & 581

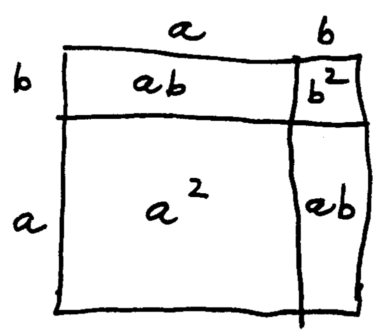
- KURT GÖDEL
  - Univ. of Vienna – 1931
  - in any system of axioms rich enough to cover arithametic ~~of~~ or Euclidean Geometry there would always be undecidable propositions
  - consistency of the axioms is one of the undecidable propositions!

THE QUADRATIC EQUATION

- In Elements algebraic problems are cast in geometric language,
  - No adequate algebraic symbolism

We write $(a+b)^2 = a^2 + 2ab + b^2$

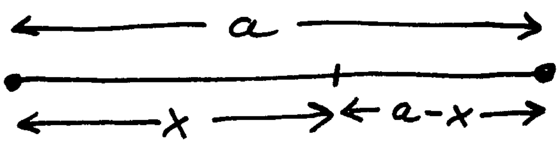
Euclid draw



$$(x+a)x = a^2$$

Kepler called this
"one of the two Jewels of Geometry"

- This divides the segment a into the
"golden section"



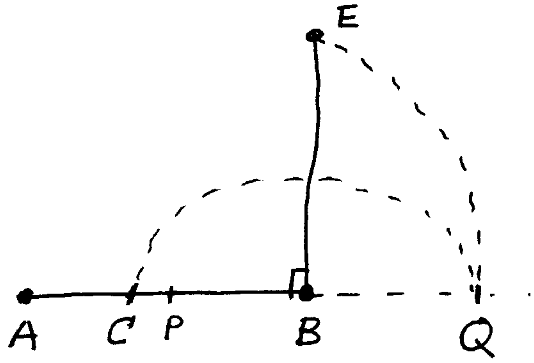
$$\frac{a}{x} = \frac{x}{a-x} \qquad x > a-x$$

$$x = \tfrac{1}{2}a(\sqrt{5}-1)$$

When $a=1$, $x = \dfrac{\sqrt{5}-1}{2}$ = reciprocal of the golden ratio

0.6180339

EUCLID'S CONSTRUCTION OF THE GOLDEN SECTION



(1) erect $\overline{BE} = \overline{AB}$

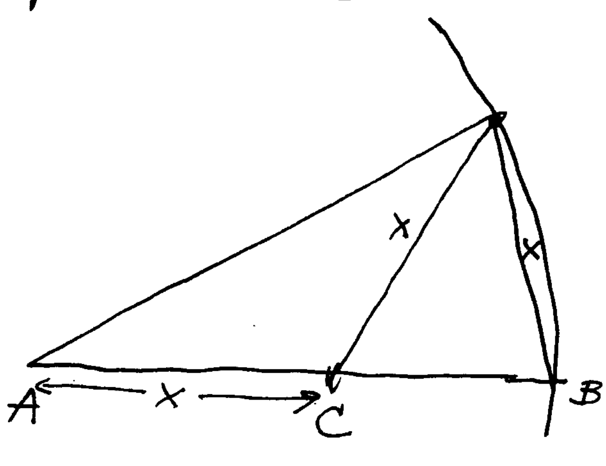
(2) FIND P = midpoint of AB

(3) WITH P AS CENTER DRAW ARC EQ, radius $\overline{PE}$

(4) WITH B AS CENTER, DRAW SEMICIRCLE OF RADIUS $\overline{BQ}$

(5) $\overline{BC}$ IS GOLDEN SECTION OF $\overline{AB}$

- Greeks were able to inscribe polygons in a circle of sides 3, 4, 5, 6, 8, 10, but not 7 sides.
- To find side length for 10 sided regular polygon, use golden section of the radius



- Western Europeans first learned algebra from works of

Muhammed ibn Musa al-Khowarizmi (820 ad)

- astronomer & math. from Bagdad

- name algebra is from al-jabr, part of title of his treatise

Hisab al-jabr w'al muquabalah

"The science of reunion and deduction"

- In 12th century this book was translated into Latin

"Algebrae et Almucabola"

Homework

p. 186 #3

EUCLID'S NUMBER THEORY

Thm There are an infinity of primes.

Proof

Assume p is the largest prime.

Then form the number

$$N = \underbrace{(2\cdot 3\cdot 5\cdot 7\cdot 11\cdot 13 \cdots p)}_{\text{product of all primes}} + 1$$

Since $N > p$, N is not prime.

However no prime q divides N since

$$\frac{N}{q} = \underbrace{\frac{(2\cdot 3\cdot 5 \cdots \cancel{q} \cdots p)}{\cancel{q}}}_{\substack{\text{whole no.}\\ \text{since } q \text{ in}\\ \text{numerator}}} + \underbrace{\frac{1}{q}}_{\substack{\text{non zero}\\ \text{fraction}}}$$

Thus N is prime contradicting the assumption that p is the largest prime.

Eratosthenes (276–194 BC)

- studied at Plato's school in Athens
- called at age 30 to Alexandria to tutor Ptolemy II 's son
- became chief librarian at Museum last 40 years of his life
  - most prestigious position in Hellenistic world
- foremost scholar of his day
  - wrote on geography, philosophy, history, astronomy, math & literary criticism

Sieve of Eratosthenes (find all primes <100)

2 ~~3~~ 3 ~~4~~ 5 ~~6~~ 7 ~~8~~ ~~9~~ ~~10~~

11 ~~12~~ 13 ~~14~~ ~~15~~ ~~16~~ 17 ~~18~~ ~~19~~ ~~20~~

~~21~~ ~~22~~ 23 ~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~ 29 ~~30~~

⋮

~~100~~

First cross out all multiples of 2 (except 2)

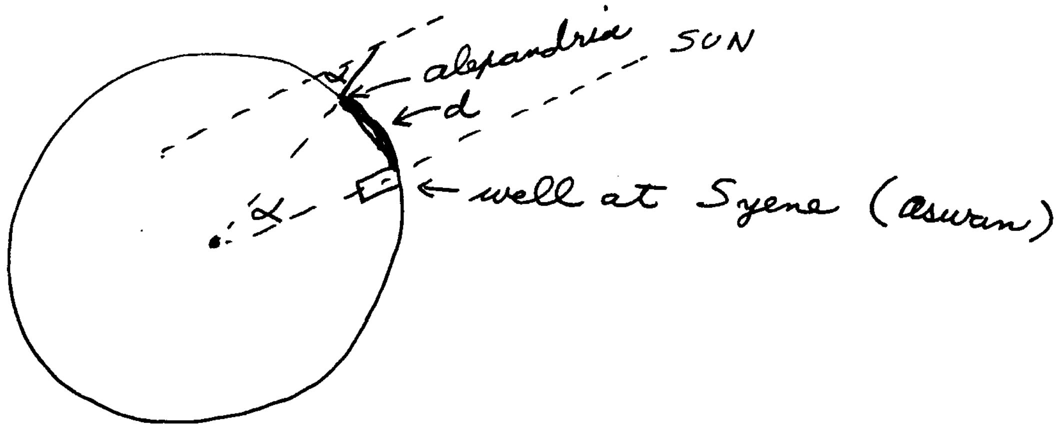
" 3 (" 3)

" 5 (" 5)

" 7 (" 7)

We can stop here since $11^2 > 100$

- best remembered for estimating earth's circumferen



$$\frac{\alpha}{360^\circ} = \frac{d}{\text{Circumference of earth}}$$

- got 24,662 miles (245 miles short)

Claudius Ptolemy (100 - 170 ad)

- did for astronomy what Euclid did for geo.
- wrote "Almagest"
  - supreme authority on astron. until Copernicus's "De Revolutionibus (1543)
  - chief flaw is earth centered