Part 2

Outline for the

History of Mathematics

Dr Osler

Start 7 (PART II (NEW VERSION SPRING 2004))

ARCHIMEDES (287-212 BC)

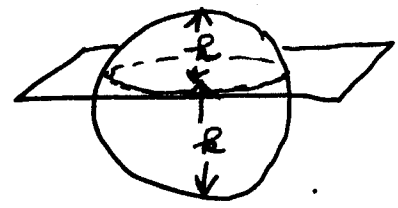
- greatest creative genius of ancient world
- lived in Syracuse
  - Greek settlement in Sicily
- worked under King Hieron II
- invented Archimedean Screw
  - simple water pump 
- was able to launch a ship of great weight using pulleys and levers
- Plutarch wrote
  "he placed his whole ambition in those speculations whose beauty and subtlety are untainted by any admixture of the common needs of life"
- Solved the problem of determining if King Hieron's crown
  - was the gold diluted with silver
- Built machines to assist in the war defense of the city of Syracuse
  - terrified Roman Soldiers
- killed by common soldier sacking the city
- inscribed on his tomb

cylinder containing sphere

Sphere = $\frac{2}{3}$ cylinder

- Archimedes was led to a cubic eq from the problem:

Cut a sphere by a plane into 2 volumes in a given ratio



r = radius

$$\frac{\text{Volume Top}}{\text{Bottom}} = \frac{m}{n} \leftarrow \text{given ratio}$$

$$\frac{\pi h^2\left(r - \frac{h}{3}\right)}{\pi k^2\left(r - \frac{k}{3}\right)} = \frac{m}{n}$$

where $h + k = 2r$

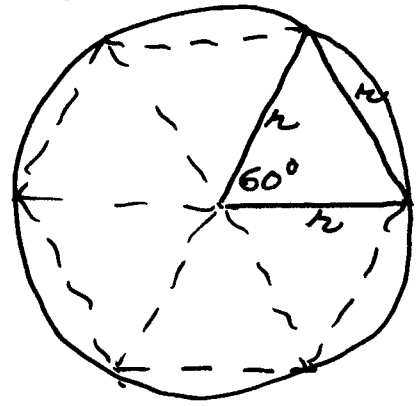
This reduces to

$$(m+n)h^3 - 3r(m+n)h^2 + 4m r^3 = 0$$

a cubic to solve for h

- Archimedes solved this as the solution of two simultaneous quadratic curves
- Cubics would not appear again in math history for over 2000 yrs

- To approximate π
  - be inscribed and circumscribed polygons of sides 6, 12, 24, 48, 96
  - to inscribe hexagon (6 sides) simply use radius as chord



Let p_n = perimeter of reg. polygon on n sides inscribed

P_n = " " "

" circumscribed

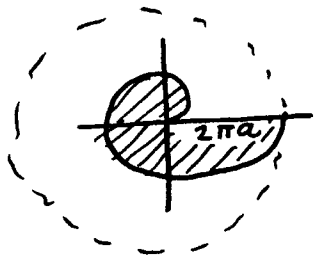
circumference

$$p_6 < p_{12} < p_{24} < \cdots p_n < C < P_n \cdots < P_{24} < P_{12} < P_6$$

- Archimedes estimated the number of grains of sand needed to fill the universe
  - invented an exponential number notation to achieve this

- Archimedean Spiral

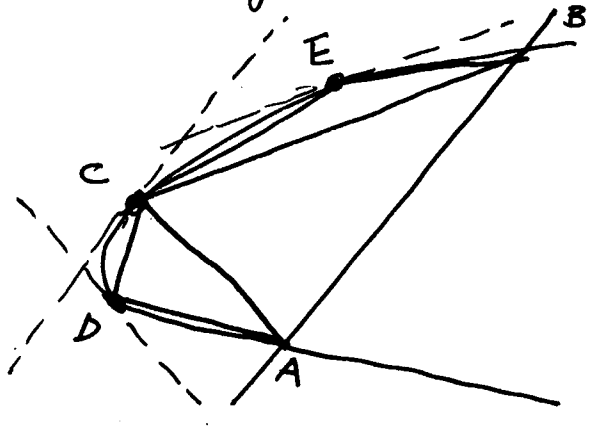
$r = a\theta$



showed that area = $\frac{1}{3}\pi(2\pi a)^2$

Homework
p. 227; 4, 5, 10

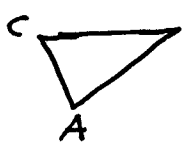
- Archimedes found area of parabola cut by a chord



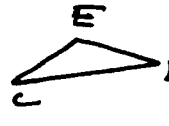
area of $ABC = \Delta$
" " $CEB = \frac{1}{8}\Delta$
" " $CDA = \frac{1}{8}\Delta$

Area of

$=$



$+ \; 2$



$+ 4$ ——— $+ \cdots$

$= \Delta + 2\left(\frac{1}{8}\Delta\right) + 4\left(\frac{1}{8^2}\Delta\right) + 8\left(\frac{1}{8^3}\Delta\right) + \cdots$

$= \Delta\left[1 + \frac{1}{4} + 2^2\left(\frac{1}{2^6}\right) + 2^3\left(\frac{1}{2^9}\right) + \cdots\right]$

$= \Delta\left[1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots\right]$

$= \Delta \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}\Delta$

About Start of Class 8

Chapter 5

The Second Alexandrian School; Diophantus

The Decline of Alexandrian Math

146 BC Ptolemy VII
- exiled scholars not loyal to him
- Alexandria no longer leading center of research
- refugee scholars bring knowledge to other lands

- from 750 BC to 450 AD
  - there is no Roman mathematician of note
  - Romans interested in practical applications

- By 4th century ad
  - great days of Greek math had passed
  - intellectual energy turns to questions of theology
  - Western Europe
    - blanketed with barbarism & illiteracy
  - Eastern Europe (Byzantine Empire)
    - preserved copies of ancient works
    - did little original work

The Arithmetica

- Diophantus
  - lived in Alexandria (about 250 ad)
  - wrote "Arithmetica"
    - earliest treatise on algebra
    - introduced symbols for unknowns

Diophantine Eqs

- letter from Archimedes to Eratosthenes has famous "cattle problem"

W = no. of bulls that are white
w = " cows " "
X = " bulls " black
x = " cows " "
Y = " bulls " spotted
y = " cows " "
Z = " bulls " brown
z = " cows " "

$W = \frac{5}{6}X + Z$, $X = \frac{9}{20}Y + Z$, $Y = \frac{13}{42}W + Z$

$w = \frac{7}{12}(X + x)$, $x = \frac{9}{20}(Y + y)$, $z = \frac{13}{42}(W + w)$

$W + X$ = square no. $Y + Z$ = triangular no.

This all reduces to

$$X^2 - 4,729,494\, y^2 = 1$$

where y is a multiple of 9314

- leads to $x^2 - a y^2 = 1$
  Pell's eq.
- many tried to solve cattle prob. but were discouraged by large nos.
- 1880 Amthor
  no. of cattle = 776 ... in 206,542 digits
- 1965 solved on computer giving all 206,545 digits

- Alexander's invasion of India
  - stimulates math
- from 400 to 1200 Indians develop a math system superior to Greeks in all but geometry
  - Aryabhata
    summed arith. & geo. series
    table of sines
    $\pi = 3,1416$
  - Brahmagupta (century after Aryabhata)
    - $\pi = \sqrt{10}$
    - negative nos.
    - had both sols of quadratic
    - solved eqs for whole nos.
    - studied indeterminate eqs.

The Later Commentators

- time of Diophantus.
  - final stages of Greek math
  - most later writers are commentators on older works
  - Pappus is an exception

Hypatia (370 - 415)
- 1st prominent woman math.
- killed by mob of religious zealots in streets

Roman period
- devoid of interest in theoretical math
- Boethius
  - early Roman commentator
  - condensed versions of
    - Euclid,
    - Nicomachus
  - middle ages learned arith thru him

Chapter 6

The First Awakening: Fibonacci

In western Europe the dark ages (5th to 11th centuries) is low ebb of math.

Arabs who overran southern & eastern shores of Mediterranean

- bring no original scholarship
- collect old manuscripts from lands they conquer
- these they translate into Arabic
- by 10th century nearly all surviving Greek texts are available in Arabic
- Arabs prevented many Greek texts from becoming lost
  - Islam's great contribution to the advancement of knowledge

- Arabs added significant material from Persia and India to the Greek
  - Hindus had interest in arithmetic & algebra
  - Most significant idea borrowed from east:
    - Arabic numerals
      - 9 digits & zero
    - Also developed trigonometry

- As soon as it was known that masterpieces of antiquity were in Arabic, many scholars undertook to translate them into Latin

- Gerard of Cremona 1114-1187
  - drawn to Toledo
    - Translated Almagest into Latin
    - also 90 other Arabic texts including Archimedes Apollonius

Abelard of Bath (1090-1150)

- traveled widely seeking knowledge
  - Spain, Italy, Sicily, Grace, Syria, Palestine
- disguised as a Mohammedan
- translated Euclid's Elements to Latin

- these struggling translators received little or no remuneration
- devoted to truth & knowledge

about start of class #9

"Liber Abaci" & "Liber Quadratorum"

Leonardo of Pisa

- Fibonacci (born 1175)
- greatest math. of middle ages
- wrote Liber Abaci in 1202 (Book of Counting)
  - explained arabic number system
    - chief way in which Europe learned of these numbers

- math masterwork of middle ages

- Origins of Hindu - Arabic numbers sys.
  - obscure & disputed
    - originated in India ? 3rd Century
    - carried to Bagdad 8th Century
    - transmitted to western Europe by way of Moorish Spain

- Resistance to spread of new numerals
  - 1299 Florence
    - ordinance forbidding merchants to record in these
    - due to great variety of shapes of digits
      ex O looks like 6 or 9

- after printing (1450)
  - digits stabalized
  - appear as today

Liber Quadratorum (1225)

- by Fibonacci
  - pioneer in revival of math
  - gave many original proofs and results
  - does not surpass work of his Arab predecessors
  - notes that Euclid's Book X classification of irrationals is not complete

- given challenge problem

$$x^3 + 2x^2 + 10x = 20$$

  - first cubic in Europe since time of Greeks

$$10\left(x + \frac{x^2}{5} + \frac{x^3}{10}\right) = 20$$

$$x + \frac{x^2}{5} + \frac{x^3}{10} = 2$$

Fibonacci showed that x cannot be rational!

assume $x = \frac{a}{b}$ where a & b are integers with no common factors.

$$\frac{a}{b} + \frac{a^2}{5b^2} + \frac{a^3}{10b^3} = \frac{a(10b^2+2ab+a^2)}{10b^3}$$

$$= 2$$

This will not be an integer unless $10b^3$ (and certainly b) divides $10b^2+2ab+a^2$. Since b divides $10b^2+2ab$, then b must divide a^2. This violates the assumption that $\frac{a}{b}$ is reduced!

Thus x is not rational!