Part 3

Outline for the

History of Mathematics

Dr Osler

Ptolemy vs Copernicus

Thomas J. Osler
Mathematics Department
Rowan University
Glassboro, NJ 08028

Osler@rowan.edu

1. Introduction

Before recorded history, it was observed that the stars formed fixed patterns in the sky. These became our constellations. They watched the sun, moon and five of the brightest stars (the planets) move through the fixed stars in a great band called the zodiac that circles the heavens. The twelve signs are the twelve constellations that comprise this zodiac. The path that the sun takes through the zodiac is called the ecliptic. The moon and planets move in paths that are very close to the ecliptic. To identify an object's position on the ecliptic, astronomers measure its distance in degrees from the first point in the constellation of Aries. In this paper we will always use the angle θ to measure this *ecliptic longitude*. The angle θ is nearly the *right ascension* of the object. The right ascension is the angle measured in *hours* and *minutes* along the celestial equator. (There are 24 hours in a full circle.)

About 150 ad, Claudius Ptolemy published his *Almagest* which was the bible of astronomy for 1500 years. In his cosmology, the earth was at the center of the universe. We will use a simplified version of Ptolemies cosmos to obtain equations for our ephemeris is section 2.

In 1543 Nicolaus Copernicus, a Polish priest, published his *De Revolutionibus* in which he placed the sun at the center.

This analysis requires nothing more than familiarity with trigonometry. Some small experience with astronomy is helpful, but not necessary. This could be used as supplemental material in a course in precalculus mathematics, elementary physics, and in an introductory computer-programming course.

2. The Simplified Ptolemaic View

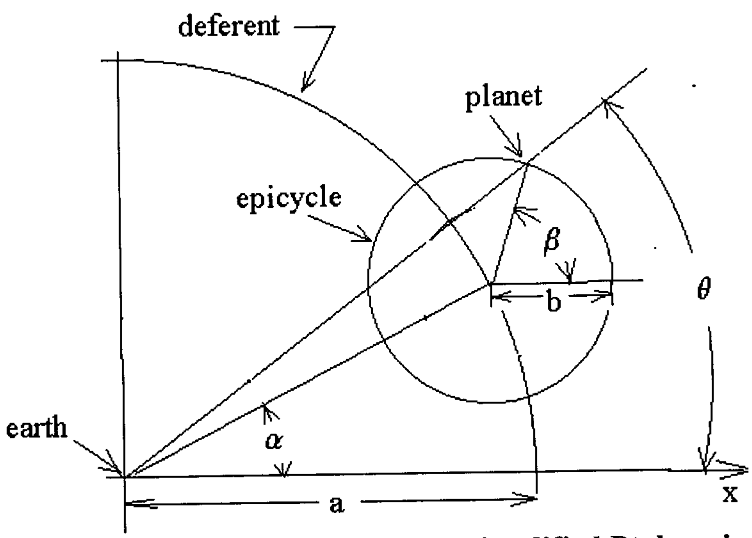


Figure 1: The simplified Ptolemaic system

Figure 1 shows the essential features of the ptolemaic system. The planet is on the rim of a small circle of radius b called the epicycle. This circle rotates with constant angular velocity ω_E. The angle β is given by

(2.1) $$\beta = \omega_E t + \gamma_E,$$

where t is time and γ_E is the value of β when $t = 0$. The center of the epicycle in turn moves uniformly on the rim of a large circle of radius a called the deferent. The angle α is given by

(2.2) $$\alpha = \omega_P t + \gamma_P.$$

The x and y coordinates of the planet are found using

(2.3) $$x = a\cos(\omega_P t + \gamma_P) + b\cos(\omega_E t + \gamma_E),$$

(2.4) $$y = a\sin(\omega_P t + \gamma_P) + b\sin(\omega_E t + \gamma_E).$$

Since the earth is at the origin of coordinates, the position of the planet is seen as the angle

(2.5) $$\theta = \tan^{-1}\left(\frac{y}{x}\right).$$

3. The Simplified Copernican View

In the Copernican view, the sun is at the center of coordinates as shown in Figure 2. The earth is orbiting the sun on a circle of radius b while the planet is on a circle of radius a. The angles α and β are again given by relations (2.1) and (2.2).

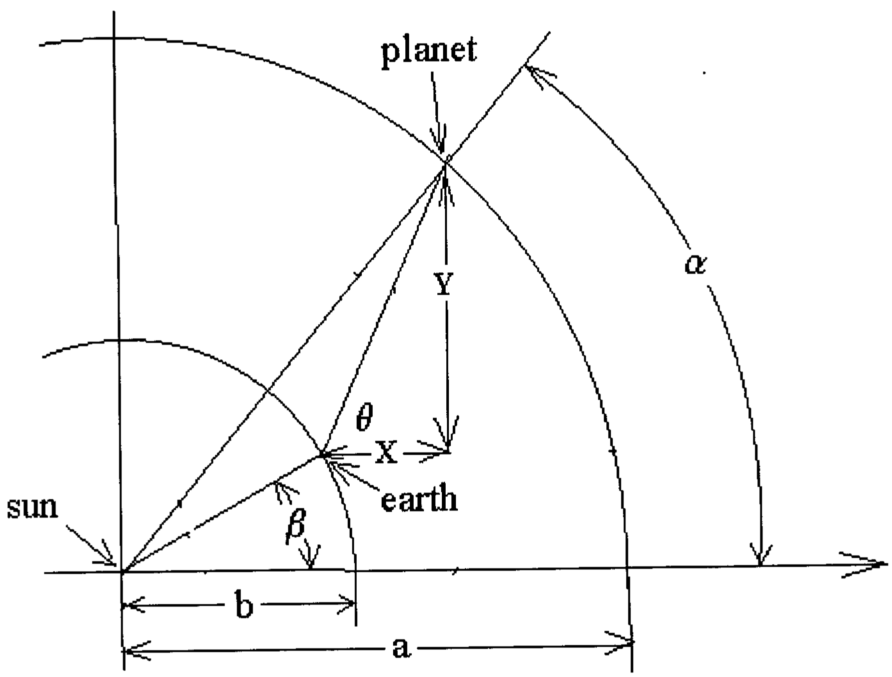


Figure 2: Simplified Copernican system

The values of X and Y are

(3.1) $$X = a\cos(\omega_P t + \gamma_P) - b\cos(\omega_E t + \gamma_E),$$ and

(3.2) $$Y = a\sin(\omega_P t + \gamma_P) - b\sin(\omega_E t + \gamma_E).$$

The angle at which the planet is observed in the heavens from the observer on the earth is given by

(3.3) $$\theta = \tan^{-1}\left(\frac{Y}{X}\right).$$

4. Ptolemy vs. Copernicus, what's the fuss?

Historically, the Copernican hypothesis was very controversial. Early supporters of the theory faced the ridicule of those who believed that a moving earth would result in everything flying off into the air. Worse, there were religious objections to moving man from the central location of God's creation. Johannes Kepler was excommunicated from the Lutheran church, and Galileo was tried by the inquisition for supporting the Copernican view.

However, from a mathematical point of view, the two theories are equivalent. Compare the Ptolemaic equations (2.3), (2.4) and (2.5) with their equivalent Copernican formulas (3.1), (3.2) and (3.3). The only difference is the minus signs in (3.2) and (3.3). If we replace γ_E by $\gamma_E + \pi$ in (3.1) and (3.2), then the minus signs disappear, and the Ptolemaic and Copernican formulas for the position of a planet become identical. The addition of π simply changes the reference angle from the positive x direction to the negative x direction. Thus from a mathematical point of view, there is no difference in the Ptolemaic and the Copernican systems.

References

[1] *The Astronomical Almanac for the Year 2001,* U. S. Government Printing Office, Washington, D.C.,

[2] *Raphael's Astronomical Ephemeris of the Planets' Places for 2001*, W. Foulsham & Co. LTD., Bennetts Close, Slough, Berks, England, 2000.

[3] Copernicus, Nicolaus, *On the Revolutions of the Heavenly Spheres,* (translated by C. G. Wallis), Vol. 16 of *Great Books of the Western World,* R. M. Huthins, editor, William Benton, Pub., Encyclopedia Britanica, Inc., Chicago, 1952, pp. 505-838.

[4] Ptolemy, Claudius, *The Almagest*, (translated by R. C. Taliaferro), Vol. 16 of *Great Books of the Western World,* R. M. Huthins, editor, William Benton, Pub., Encyclopedia Britanica, Inc., Chicago, 1952, pp. 5-465.

[5] Teets, Donald and Whitehead, Karen, C*omputation of planetary orbits,* The College Mathematics Journal, 29(1998), pp 397-404.

[6] Teets, Donald and Whitehead, Karen, *The discovery of Ceres: How Gauss became famous,* Mathematics Magazine, 72(1999), pp. 83-93.

[5] Teets, Donald and Whitehead, Karen, C*omputation of planetary orbits,* The College Mathematics Journal, 29(1998), pp 397-404.

[6] Teets, Donald and Whitehead, Karen, *The discovery of Ceres: How Gauss became famous,* Mathematics Magazine, 72(1999), pp. 83-93.

CHAP 8

THE MECHANICAL WORLD: DESCARTES & NEWTON

DAWN OF MODERN MATH

- By late 1600's
  - scientific, technological & economic leadership centered on the English Channel
  - eagerness to experiment
    - William Gilbert's "De Magnete" 1600
      - 1st treatise based entirely on experimentation
    - half-century from 1637 to 1687 is "fountainhead of math"
      - 1637 – Descartes's "La Geometria"
      - 1687 – Newton "Principia Mathematica"
    - Fermat & Descartes
      - wed algebra & geometry ⇒ analytic geometry

John Napier (1550-1617)

- invented logarithms
  - one of greatest computational improvements in arith.
  - $\times$ & $\div$ reduced to $+$ & $-$
  - "by shortening the labors, doubled the life of the astronomer" - Laplace

- Rise of calculus

  "If I have seen farther than others, it is because I have stood on the shoulders of giants" - Newton

- Galileo Galilei (1564-1642)
  - associated with Copernican revolution
  - birth of modern science
  - dethronment of Aristotle
  - struggle against external restrictions on scientific enquiry

- Droped 2 unequal wts from Leaning tower of Pisa to disprove Aristotle

Galileo (cont)

- first to publish telescopic astronomical observations
  - discovered 4 satellites of Jupiter
    - disproof of Aristotelian view that earth is center of all astro. motions

- at start of 16th century, most believed in geo-centered universe
  - earth center
    - 9 crystalline spheres
      1. sun
      2. moon
      3-7 - planets
      8. stars
      9. god (primum mobile)
  - earth is central - everything exists for it

Nicolaus Copernicus (1473-1543)

- sun ~~changes f~~ is center
  - theologians both Catholic & protestant dislike this

- Galileo finds himself in disputes about his astro. views & Bible
    - Math is denounced from pulpit as a devilish art and all mathematicians as enemies of the true religion
    - Church forbids him to lecture on Copernican theory ~~[illegible]~~
- In 1623 a new Pope
    - Urban VII
        - admired Galileo
        - granted " permission to publish
    - Galileo write "Dialogue"
        - in Italian not Latin
            - reach wide audience
        - make aristotelian view seem absurd
- at age 70 Galileo is summoned to stand trial in Rome by Inquisition
    - sentenced to house arrest

- Galileo like Plato
  - was maker of mathematicians rather than the author of math treatises
  - his great contribution:
    - math is vehicle for scientific explanation

- Johannes Kepler (1571 - 1630)
  - born near south Germany
  - poverty
  - father a drunkard
  - mother tried as witch
  - Left his studies to be a Luthern minister since the church was opposed to Copernican view
  - Lectured on Math
    - beyond capacity of his students
      - 1st yr 12 students
      - 2nd yr none

Keplers idea

Saturn

CUBE

Jupiter

TETRAHEDRON

Mars

DODECAHEDRON

Earth

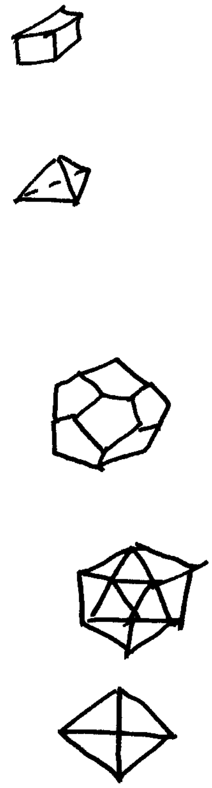
ICOSAHEDRON

Venus

OCTOHEDRON

Mercury

• SUN



~~spheres are inscribed & cir~~

planetary spheres are inscribed & circumscribed by five regular solids

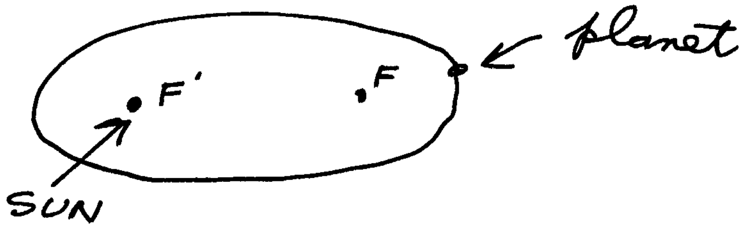
TYCHO BRAHE (1546-1601)

- Danish astronomer
- made very accurate observations
- invited Kepler to work with him
- when he died, Kepler inherited his data

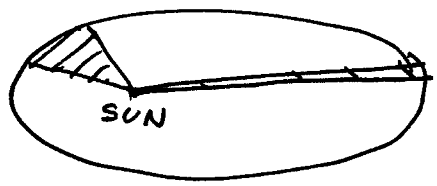
- riddle of orbit of Mars
  - led Kepler to see that planets moved in elliptical, not circular orbits.

- Kepler's 3 laws of planetary motion

(1)



(2)



sweep out equal areas in equal times

(3) Relates Period (yr) to semi-major axis of orbit

Descartes: The Discours de la Methode

René Descartes 1596–1650

- marks turning point between medieval and modern mathematics
- math only subject he found satisfying at an early age
  - because of its demonstrations
- health delicate
  - allowed to lie in bed each morning as late as he pleased
    - continued doing so all his life
- Nov 10, 1619
  - dreamed 3 feverish dreams
    - discovered foundations of a marvelous science
- 1629 to 1633
  - built up a cosmological theory of vortices
    - called Le Monde
      - he failed to publish it since he was a devout catholic & did not want to offend the church

Discours de la Methode ---

- 1637
- most significant of Descartes' writings
- philosophy of systematic doubt
  - seeks to use math demonstrations in all disciplines

La Geometrie

- 1664
- invention of Analytic Geo.
  - joint algebra & geo.
- The Tangent Problem
  - led to it by his optical studies
    - quote " I dare say that this is not only the most useful and the most general problem in geometry that I know, but even that I have ever desired to know.

– How Descartes found tangents

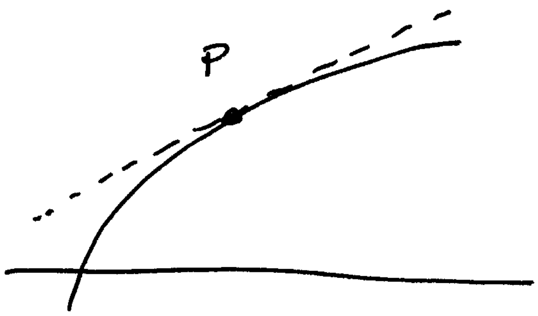


FIG 1 – The problem.
Find the tangent at P to $f(x,y)=0$

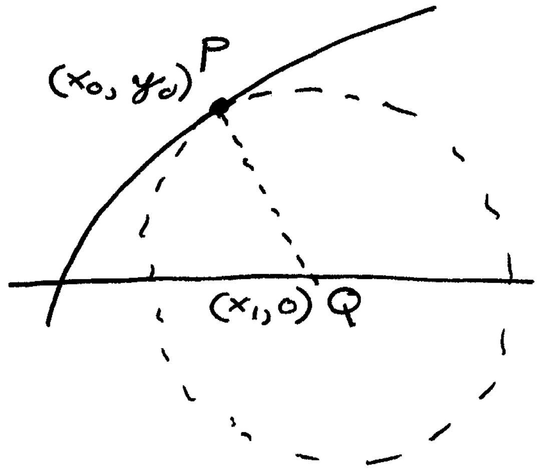


FIG 2 – THE CIRCLE center at Q on x-axis just touching the curve $f(x,y)=0$ at P.

EQ. OF THE CIRCLE

(1) $$(x-x_1)^2 + y^2 = (x_0 - x_1)^2 + y_0^2$$

NEXT eliminate y between (1) and $f(x,y)=0$ to get $g(x, x_1) = 0$.

Normally a circle intersects a curve in 2 pts.

But ours only intersects in one.

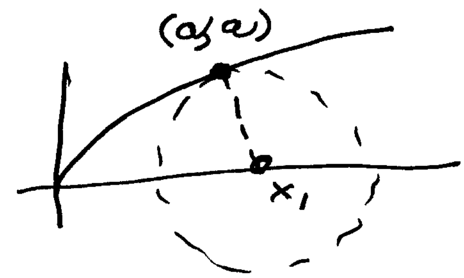
Thus we effect a "double root" for x in $g(x, x_1) = 0$. Now select x_1 so that a double route occurs.

- (x_0, y_0) & $(x_1, 0)$ define a normal line at P
  - from this we can get the tangent at P.

EXAMPLE OF DESCARTES' METHOD

$f(x,y) = 0$ will be $y^2 - ax = 0$

$P: (x_0, y_0)$ will be (a, a)



Eq of circle is

$$(x-x_1)^2 + y^2 = (a-x_1)^2 + a^2$$

To eliminate y set $y^2 = ax$

$$(x-x_1)^2 + ax = (a-x_1)^2 + a^2$$

$$x^2 - 2x_1 x + \cancel{x_1^2} + ax = a^2 - 2ax_1 + \cancel{x_1^2} + a^2$$

$$x^2 + (a - 2x_1)x + 2a(x_1 - a) = 0$$

For this relation to have a double root we know that

$$B^2 = 4AC \quad \text{or}$$

$$(a - 2x_1)^2 = 8a(x_1 - a)$$

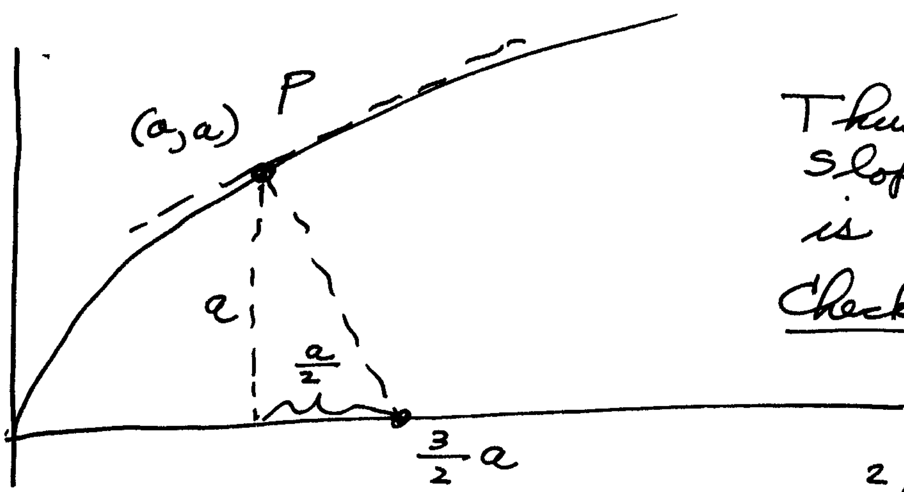
$$a^2 - 4ax_1 + 4x_1^2 = 8ax_1 - 8a^2$$

$$4x_1^2 - 12ax_1 + 9a^2 = 0$$

$$(2x_1 - 3a)^2 = 0$$

$$2x_1 = 3a$$

$$x_1 = \frac{3}{2}a$$



Thus the slope at P is $\frac{1}{2}$

Check:

$$y^2 = ax$$

$$2yy' = a$$

$$y' = \frac{a}{2y}$$

$$y'\Big|_{a=y} = \frac{1}{2}$$

ok

DESCARTES'S RULE OF SIGNS

- from 3rd & last book of *La Géométrie*

- The no. of positive roots (each counted as often as its multiplicity) of a polynomial with real coef. is either equal to the no. of variations in sign, or this no. decreased by a positive even integer.

Ex

$$x^3+x^2-x+2=0$$

has either 2 positive roots or none.

- The negative roots of $f(x)=0$ are the positive roots of $f(-x)=0$.

<u>Ex</u>

$$x^6 - 10x^2 + x + 1 = 0$$

replace x by $-x$ to get

$$x^6 - 10x^2 - x + 1 = 0.$$

This eq has 2 changes of sign. Thus there are no more than 2 neg. roots. (Also no more than 2 pos.)

Thus at least 2 complex roots

<u>HW</u> p. 359

4, 6, 7, 8, 9

NEWTON: The Principia Mathematica

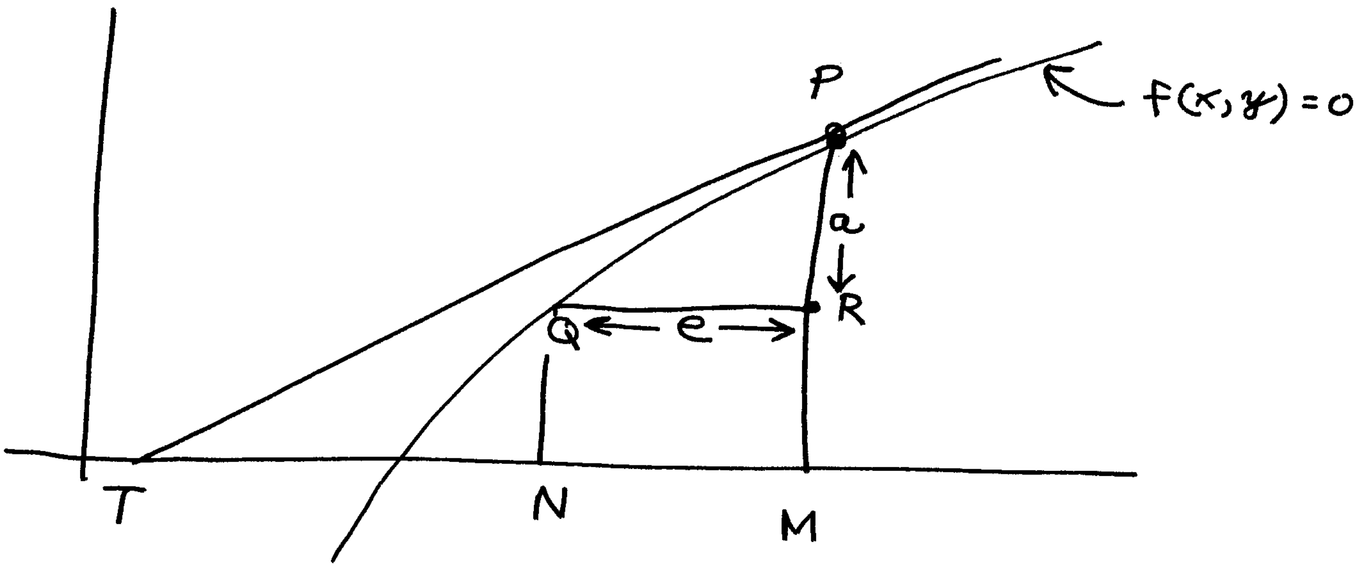
- Galileo died
  - Christmas day 1642
- Newton is born 1642
  - father died before he was born
  - born premature
    - not expected to live but a few days
  - lived to be 85 yrs old
- was not a precocious child

Isaac Barrow (1630-1677)

- 1st occupant of Lucasian chair
- inspired Newton
- early member of Royal Society
- very nearly invented calculus

Barrow's Method of Finding Tangents

– from Lectiones Geometricae



If triangle PQR is very small, then

$\triangle PQR$ is similar to $\triangle PTM$

thus $\quad \dfrac{TM}{MP} = \dfrac{QR}{RP} = \dfrac{e}{a}$

Point P has coordinates (x, y)
" Q " " $(x-e,\ y-a)$

Next simplify $f(x-e,\ y-a) = 0$
"reject all terms in which there is no <u>a</u> or <u>e</u> (for they destroy each other by the nature of the curve); reject all terms in which <u>a</u> and <u>e</u> are above the first

power, or are multiplied together (for they are no value with the rest, as being infinitely small)".

EXAMPLE

$$f(x,y) = x^3 + y^3 - r^3 = 0$$

$$f(x-e, y-a) = (x-e)^3 + (y-a)^3 - r^3 = 0$$

$$\underline{x^3} - 3x^2e + \underline{\underline{3xe^2}} - \underline{\underline{e^3}} + \underline{y^3} - 3y^2a + \underline{\underline{3ya^2}}$$
$$\underline{\underline{-a^3}} \ \underline{-r^3} = 0$$

① reject all terms in which there is no $\underline{a}$ or $\underline{e}$ ($\underline{\ \ }$)

② reject all terms in which $\underline{a}$ and $\underline{e}$ are above the first power ($\underline{\underline{\ \ }}$)

We are left with

$$-3x^2e - 3y^2a = 0$$

$$x^2e + y^2a = 0$$

$$y^2a = -x^2e$$

$$\boxed{\frac{a}{e} = -\frac{x^2}{y^2}}$$

NEWTON'S INVENTION OF BINOMIAL THM.

- pass out 3 pages for 17th century text
- tell about my finding these in 1966 at St Joseph's College library
- my shock at how it differs from modern calculus
  - no product, quotient, etc. rules
  - uses only power rule
- how my own discovery of D^α is related to this

NEWTON'S INVENTION OF THE BINOMIAL THEOREM.

REVIEW

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2+2ab+b^2$

$\vdots$

$(a+b)^5 = a^5+5a^4b+\cdots$

- Show how to get $\binom{p}{r}$ from $\binom{p-1}{r-1}+\binom{p-1}{r}$
- How would you do

$(a+b)^{100} =$

if you did not have $(a+b)^{99}$, This is the problem Newton faced.

Source He derived those ingenious and useful Hints, that were afterwards improved by him, into the noblest Inventions and most sublime Theories. I am very sensible, that for his singular Sagacity this way He was much indebted to Nature. But I am apt to believe most Persons are endued with a good share of it; such I mean as have a Capacity and Genius for Mathematical Speculations. And I am persuaded, that our great Admiration of others, whom we see eminently conspicuous for this Talent, arises more from our own Neglect to cultivate the Bounty of Nature to us, than from any extraordinary Difference there is in her Gift.

This naturally leads me to the last Thing I shall observe on this Head. And that is our Author's unwearied Diligence and incessant Application in improving the Hints and Conceptions he had once formed, to the highest Perfection. He pusheth his Invention through all the Difficulties that can arise, extends it to all the Varieties of Cases that can happen, and at last applies it to the most curious Purposes.

These distinguishing Excellencies in this elementary Treatise of the greatest Master of Mathematical Learning that perhaps ever appeared in the World, I thought it not amiss to mention in this Place: As I conceive they afford the strongest Motives and most powerful Incitements to His Disciples, to follow their Great Leader in those Steps, by which He attained to the highest pitch of Human Glory.

I shall now proceed to take a short View of the Body of the Work, which may be divided into these two principal Members. The first is, the Method of Fluxions in its general Sense; and the other is the Application of this Method to the Geometry of Curve Lines. The first of these may again be subdivided into two Parts. The first of which contains the Doctrine of infinite Series, and is an Introduction to the other, wherein is delivered the Method of Fluxions peculiarly so called.

In

In the first Part or the Method of Infinite Series, the Author very much enlarges the Boundaries of Analyticks by introducing into Algebra or specious Arithmetick a new Way of expressing Universal Radicals, (such as $\sqrt{rr-cc}$*,) by an infinite Series of simple Terms, which continually approach towards the true Value of the Roots, and if infinitely continued will be equal to them, and therefore may be used instead of them. He begins with pointing out the particular Analogy that would, if attended to, naturally furnish the Hint for this Improvement,* viz. *the Conformity there is between the Relation of Decimal Fractions to Vulgar Arithmetick, and that of Infinite Series to Common Algebra; and He explains the manner of this Correspondence. He has not given us here the particular Occasion which led him into the Road of improving the Hint. This was beside his present Purpose. But because, as I have already observed, a very good Use may be made of such Histories, and especially as this is communicated by the Author himself *, I shall therefore give it the Reader in* English *as follows.*

'*Not long after I had entered upon the Study of the* '*Mathematicks, whilst I was perusing the Works of* '*Our Celebrated Dr.* Wallis, *and considering the Series (of Universal Roots) by the Interpolation of which* '*He exhibits the Area of the Circle and Hyperbola, for* '*instance, in this Series of Curves whose Base or common Axis call* $=x$*, and the successive Ordinates call* ' $\overline{1-xx}|^{\frac{0}{2}}.\ \overline{1-xx}|^{\frac{1}{2}}.\ \overline{1-xx}|^{\frac{2}{2}}.\ \overline{1-xx}|^{\frac{3}{2}}.\ \overline{1-xx}|^{\frac{4}{2}}.\ \overline{1-xx}|^{\frac{5}{2}}$, ' &c. *I observed that if the Areas of the Alternate* '*Curves which are* x, $x-\frac{1}{3}x^3$, $x-\frac{2}{3}x^3+\frac{1}{5}x^5$, $x-\frac{3}{3}x^3+\frac{3}{5}x^5-\frac{1}{7}x^7$, &c. *could be interpolated, we should* '*by this means obtain the Areas of the intermediate* '*ones; the first of which* $\overline{1-xx}|^{\frac{1}{2}}$ *is the Area of the* '*Circle.*

* Vid. Lib. supra cit. pag. 143.

In

'In order to this, first it was obvious that in each 'of these Series the first Term was x; that the second 'terms $\frac{0}{3}x^3$, $\frac{1}{3}x^3$, $\frac{2}{3}x^3$, $\frac{3}{3}x^3$, &c. were in an Arithmetical Progression, and consequently the two first 'Terms of the Series to be interpolated must be $x - \frac{\frac{1}{2}x^3}{3}$, $x - \frac{\frac{3}{2}x^3}{3}$, $x - \frac{\frac{5}{2}x^3}{3}$, &c.

'Now for the Interpolation of the rest, I considered 'that the Denominators 1, 3, 5, 7, &c. were (in all 'of them) in Arithmetical Progression, and consequently 'the whole Difficulty consisted in finding out the numeral 'Co-efficients. But these in the alternate Areas, which 'are given, I observed were the same with the Figures 'of which the several ascending Powers of the Number '11 consist, viz. 11^0, 11^1, 11^2, 11^3, 11^4, &c. 'that is first 1; the second 1,1; the third 1,2,1; the 'fourth 1,3,3,1; the fifth 1,4,6,4,1, &c.

'I applied myself therefore to seek for a Method by 'which the two first Figures of these Series might be 'derived from the rest; and I found, that if for the second Figure or numeral term we put m, the rest of 'the terms will be produced by the continual Multiplication of the Terms of this Series $\frac{m-0}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5}$, &c.

'For instance; Let the second Term m be put equal 'to 4, and there will arise $4 \times \frac{m-1}{1}$, that is 6; which 'is the third Term. The fourth Term will be $6 \times \frac{m-2}{3}$, 'that is 4. $4 \times \frac{m-3}{4} = 1$, is the fifth Term; and the 'sixth is $4 \times \frac{m-4}{1} = 0$. Which shews the Series is here 'terminated in this Case.

'This being found I applied it as a Rule to interpolate the abovementioned Series. And since in 'the Series which will express the Circle, the second

term

'term was found to be $\frac{\frac{1}{2}x^3}{3}$. Therefore I put m '$=\frac{1}{2}$, and there was produced the Terms $\frac{1}{2} \times \frac{\frac{1}{2}-1}{2}$ or '$-\frac{1}{8}$; $-\frac{1}{8} \times \frac{\frac{1}{2}-2}{3}$ or $+\frac{1}{16}$; $+\frac{1}{16} \times \frac{\frac{1}{2}-3}{4}$ or $-$ '$\frac{5}{128}$, and so on in infinitum. Hence I discovered 'that the Area sought of the Segment of the Circle is '$x - \frac{\frac{1}{2}x^3}{3} - \frac{\frac{1}{8}x^5}{5} - \frac{\frac{1}{16}x^7}{7} - \frac{\frac{5}{128}x^9}{9}$, &c.

'In the same manner the Areas to be interpolated of 'the other Curves might be produced, as might also the 'Area of the Hyperbola and of the rest of the alternate 'Curves in this Series, $\overline{1+xx}|^{\frac{0}{2}}$, $\overline{1+xx}|^{\frac{1}{2}}$, $\overline{1+xx}|^{\frac{2}{2}}$, '$\overline{1+xx}|^{\frac{3}{2}}$, &c.

'By the same Method likewise other Series might be 'interpolated, and that too if they should be taken at 'the distance of two or more intervals.

This was the way by which I first opened an Entrance into these Speculations, which I should not have 'remembred, but that in turning over my Papers a few 'Weeks ago, I accidentally cast my Eyes upon those relating to this Matter.

'When I had proceeded thus far, it immediately 'occurred to me, that the Terms $\overline{1-xx}|^{\frac{0}{2}}$, $\overline{1-xx}|^{\frac{2}{2}}$, '$\overline{1-xx}|^{\frac{4}{2}}$, $\overline{1-xx}|^{\frac{6}{2}}$, &c. that is 1, $1-xx$, $1-2xx$ '$+x^4$, $1-3xx+3x^4-x^6$, &c. might be interpolated 'in the same manner as I had done the Areas generated 'by them, and for this there needed nothing else, but only 'to leave out the Denominators 1, 3, 5, 7, &c. in the 'Terms that express the Areas; that is, the Co-efficients 'of the Terms of the Quantity to be interpolated ($\overline{1-xx}|^{\frac{1}{2}}$, 'or $\overline{1-xx}|^{\frac{3}{2}}$; or universally $\overline{1-xx}|^{m}$,) will be obtained by

a the

'the continual multiplication of the terms of this Series
'$m \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}$, &c.

'Thus (for Example) $\overline{1-xx}|^{\frac{1}{2}} = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6$, '&c. and $\overline{1-xx}|^{\frac{3}{2}} = 1 - \frac{3}{2}x^2 + \frac{3}{8}x^4 + \frac{1}{16}x^6$, &c. and '$\overline{1-xx}|^{\frac{1}{3}} = 1 - \frac{1}{3}xx - \frac{1}{9}x^4 - \frac{5}{81}x^6$, &c.

'Thus I discovered a general Method of reducing 'Radicals into Infinite Series by the Rule * which I sent 'in my last Letter, before I observed that the same thing 'might be obtained by the Extraction of Roots.

But after I had found out that method, this other way 'could not remain long unknown; for in order to prove 'the Truth of these Operations, I multiplied $1 - \frac{1}{2}x^2$ '$- \frac{1}{8}x^4 - \frac{1}{16}x^6$, &c. into itself, and the product is '$1 - xx$, all the Terms after these in infinitum vanishing; and so $1 - \frac{1}{3}xx - \frac{1}{9}x^4 - \frac{5}{81}x^6$, &c. twice drawn 'into itself produced $1 - xx$. As this was a certain 'Demonstration of the Truth of these Conclusions, so I 'was thereby naturally led to try the Converse of it, 'viz. whether these Series that now were known to be 'the Roots of the Quantity $1 - xx$ might not be extracted 'thence by the Rule for Extraction of Roots in Arithmetick; and upon trial I found it succeed to my Desire.

'I shall here set down the form of the Process in Quadraticks.

$$
\begin{array}{l}
1 - xx\left(1 - \frac{1}{2}xx - \frac{1}{8}x^4 - \frac{1}{16}x^6, \text{ \&c.}\right. \\
\underline{1} \\
0 - xx \\
\underline{-xx + \frac{1}{4}x^4} \\
\quad - \frac{1}{4}x^4 \\
\quad \underline{- \frac{1}{4}x^4 + \frac{1}{8}x^6 + \frac{1}{64}x^8} \\
\qquad - \frac{1}{8}x^6 - \frac{1}{64}x^8, \text{ \&c.}
\end{array}
$$

'This being found I laid aside the Method of Interpolation, and assumed these Operations as a more genuine Foundation to proceed upon. In the mean time 'I was not ignorant of the Way of Reduction by Division, which was so much easier.

'Proceeding upon this Foundation, the next thing I 'attempted, was the Resolution of affected Equations; 'which I also obtained, &c.

* He means the famous *Binomial* Theorem, since well known.

We have in this Account the Origin of the several Improvements the Author made in the new Way of Notation by Infinite Series: the several Branches of which are here disposed in Order and methodically digested. He first shews how to resolve by Division Fractions with multinomial Denominators. Then He proceeds to extract the Roots of Pure Powers; and lastly exhibits the Method for extracting those likewise of affected Equations. And whereas the Methods delivered before by Vieta, Oughtred, *and others, for this Operation in Numbers, were very intricate and tedious, He here supplies one much more easy and free from that Load of superfluous Terms with which theirs were incumbred.*

The Foundation being thus laid, He passeth on to the Method of Fluxions. This is the Body and principal Part of the Work. It is the distinguishing Character of our Author, that from a few plain and obvious Principles He deduceth the most surprising Conclusions; and this Part of His Character no where appears to greater Advantage than in the Invention of His Method of Fluxions. The Ancients had considered the Area of a Rectangle as produced by the Motion of one of its Sides along the other. Our Author extends this Principle to all Kinds of mathematical Quantities. The Conception is very easy and natural: We see by continual Experience that all Kinds of Figures are actually described by the Motion of Bodies. But it is evident, that Quantities generated in this manner in a given Time become greater or less, in Proportion as the Velocity with which they are generated is greater or less. These were the Considerations that led the Au-

a 2

metrical we may use in determining and demonstrating any other thing that is also geometrical.

It may also be objected, that if the ultimate ratios of evanescent quantities are given, their ultimate magnitudes will be also given: and so all quantities will consist of indivisibles, which is contrary to what Euclid has demonstrated concerning incommensurables, in the tenth book of his *Elements*. But this objection is founded on a false supposition. For those ultimate ratios with which quantities vanish are not truly the ratios of ultimate quantities, but limits towards which the ratios of quantities decreasing without limit do always converge; and to which they approach nearer than by any given difference, but never go beyond, nor in effect attain to, till the quantities are diminished *in infinitum*. This thing will appear more evident in quantities infinitely great. If two quantities, whose difference is given, be augmented *in infinitum*, the ultimate ratio of these quantities will be given, namely, the ratio of equality; but it does not from thence follow, that the ultimate or greatest quantities themselves, whose ratio that is, will be given. Therefore if in what follows, for the sake of being more easily understood, I should happen to mention quantities as least, or evanescent, or ultimate, you are not to suppose that quantities of any determinate magnitude are meant, but such as are conceived to be always diminished without end.

8.3.9

97

SECTION II

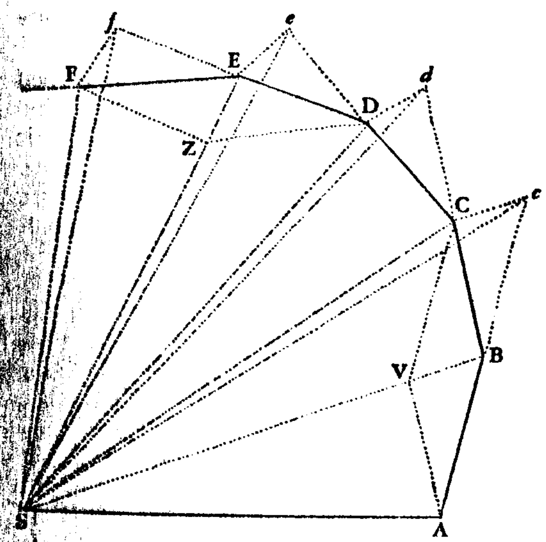
THE DETERMINATION OF CENTRIPETAL FORCES

PROPOSITION 1. THEOREM 1

The areas which revolving bodies describe by radii drawn to an immovable centre of force do lie in the same immovable planes, and are proportional to the times in which they are described.

For suppose the time to be divided into equal parts, and in the first part of that time let the body by its innate force describe the right line AB. In the second part of that time, the same would (by Law I), if not hindered, proceed directly to c, along the line Bc equal to AB; so that by the radii AS, BS, cS, drawn to the centre, the equal areas ASB, BSc, would be described. But when the body is arrived at B, suppose that a centripetal force acts at once with a great impulse, and, turning aside the body from the right line Bc, compels it afterwards to continue its motion along the right line BC. Draw cC parallel to BS, meeting BC in C; and at the end of the second part of the time, the body (by Cor. I of the Laws) will be found in C, in the same plane with the triangle ASB. Join SC, and, because SB and Cc are parallel, the triangle SBC will be equal to the triangle SBc, and therefore also to the triangle SAB. By the like argument, if the centripetal force acts successively in C, D, E, &c., and makes the body, in each single particle of time, to describe the right lines CD, DE, EF, &c., they will all lie in the same plane; and the triangle SCD will be equal to the triangle SBC, and SDE to SCD, and SEF to SDE. And therefore, in equal times, equal areas are described in one immovable plane: and, by composition, any sums SADS, SAFS, of those areas, are to each other as the times in which they are described. Now let the number of those triangles be aug-

mented, and their breadth diminished *in infinitum*; and (by Cor. IV, Lem. 3) their ultimate perimeter ADF will be a curved line: and therefore the centripetal force, by which the body is continually drawn back from the tangent of this curve, will act continually; and any described areas SADS, SAFS, which are always proportional to the times of description, will, in this case also, be proportional to those times.



Q.E.D.

COR. I. The velocity of a body attracted towards an immovable centre, in spaces void of resistance, is inversely as the perpendicular let fall from that centre on the right line that touches the orbit. For the velocities in those places A, B, C, D, E, are as the bases AB, BC, CD, DE, EF, of equal triangles; and these bases are inversely as the perpendiculars let fall upon them.

COR. II. If the chords AB, BC of two arcs, successively described in equal times by the same body, in spaces void of resistance, are completed into a parallelogram ABCV, and the diagonal BV of this parallelogram, in the position which it ultimately acquires when those arcs are diminished *in infinitum*, is produced both ways, it will pass through the centre of force.

COR. III. If the chords AB, BC, and DE, EF, of arcs described in equal times, in spaces void of resistance, are completed into the parallelograms ABCV, DEFZ, the forces in B and E are one to the other in the ultimate ratio of the diagonals BV, EZ, when those arcs are diminished *in infinitum*. For the motions BC and EF of the body (by Cor. 1 of the Laws) are compounded of the motions Bc, BV, and Ef, EZ; but BV and EZ, which are equal to Cc and Ff, in the demonstration of this Proposition, were generated by the impulses of the centripetal force in B and E, and are therefore proportional to those impulses.

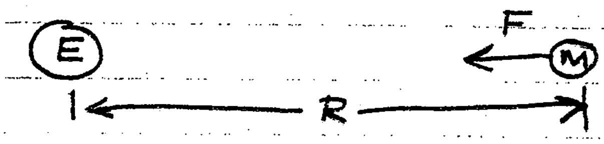
COR. IV. The forces by which bodies, in spaces void of resistance, are drawn back from rectilinear motions, and turned into curvilinear orbits, are to each other as the versed sines of arcs described in equal times; which versed sines tend to the centre of force, and bisect the chords when those arcs are diminished to infinity. For such versed sines are the halves of the diagonals mentioned in Cor. III.

COR. V. And therefore those forces are to the force of gravity as the said versed sines to the versed sines perpendicular to the horizon of those parabolic arcs which projectiles describe in the same time.

COR. VI. And the same things do all hold good (by Cor. V of the Laws) when the planes in which the bodies are moved, together with the centres of force which are placed in those planes, are not at rest, but move uniformly forwards in right lines.

EARTH – MOON SYSTEM

A TRIAL FOR NEWTON'S IDEAS.



ASSUME (1) $F = \dfrac{G m_e m_M}{R^2}$

WE CAN DERIVE F FOR CIRCULAR MOTION AS

$$(2) \quad F = m_M \frac{v^2}{R} = m_M \omega^2 R$$

ANGULAR VEL.

EQUATING (1) & (2)

$$\frac{G m_e m_M}{R^2} = m_M \omega^2 R$$

$$(3) \quad \omega^2 = \frac{G m_e}{R^3}$$

WE CAN CALCULATE $G m_e$ SINCE THE ACCELERATION AT EARTHS SURFACE IS KNOWN

$$\frac{G m_e}{r_e^2} = g \Rightarrow G m_e = g r_e^2$$

EARTH'S RADIUS

NOW (3) BECOMES

$$\omega^2 = \frac{g\, r_e^2}{R^3}$$

r_e = 4000 MILES = 21,120,000 FT
= 2.1×10^7 FT

R = 240,000 MILES = 1,267,200,000 FT
= 1.2672×10^9 FT

g = 32 FT/SEC^2

$$\omega^2 = 69.3511 \times \frac{10^{14}}{10^{27}} = 6.93511 \times 10^{-12}$$

$$\omega = 2.63346 \times 10^{-6} \text{ rad/sec}$$

= .0362127 REV/DAY

= 27.6 DAYS/REV.

GOTTFRIED LEIBNIZ: The Calculus Controversy

- invention of calculus
  - by two men
    - Newton in England
    - Leibniz on continent
  - gave rise to long & bitter controversy

- Gottfried Wilhelm Leibniz (1646-1716)
  - precocious child
  - denied degree of doctor of law at Leipzig in 1666 because he was too young
  - received doctorate next year at Altdorf
    - offered professorship on strength of his dissertation
      - choose career as a diplomat instead

Leibniz in Paris 1672-1676

- guided by Huygens in math studies

- Huygens gave Leibniz this test problem:

Find sum of $\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \cdots + \frac{1}{t_n} + \cdots$

$$t_n = \frac{n(n+1)}{2}$$

Solution:

write $\frac{2}{n(n+1)} = 2\left(\frac{1}{n} - \frac{1}{n+1}\right)$

$$\text{Sum} = 2\left[\frac{1}{1} - \frac{1}{2}\right] + 2\left[\frac{1}{2} - \frac{1}{3}\right] + 2\left[\frac{1}{3} - \frac{1}{4}\right] + \cdots$$

$$= 2\left[\frac{1}{1} \cancel{- \frac{1}{2} + \frac{1}{2}} \cancel{- \frac{1}{3} + \frac{1}{3}} \cancel{- \frac{1}{4} + \frac{1}{4}} - \cdots\right.$$

$$= 2$$

- Leibniz constructed a working model of a calculating machine that could $+, -, \times, \div$

- 1673 in London for 2 months
  - on diplomatic mission
  - elected to Royal Society
    - on exhibiting his calculating machine & due to his friend Oldenburg

- on return to Paris
  - found that he could not read a book of Huygens
    - studied Descartes' Géométrie
    - manuscripts of Pascal
  - in a few years he went from beginner to a mature mathematician

- 1673 Leibniz finds

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

(Known to Gregory in 1671)

Since this converges slowly, Newton sent Leibniz

$$\frac{\pi}{2\sqrt{2}} = 1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \frac{1}{13} - \frac{1}{15} + \cdots$$

— Leibniz invents

$\int$ = S squashed

dx, dy notations

Newton used $\dot{y}$ for $\frac{dy}{dx}$

$\ddot{y}$ " $\frac{d^2y}{dx^2}$

- 1677 invents product & quotient rules

— Leibniz never founded his calculus on the modern limit concept

- 1754 Jean d'Alembert expressed the notion

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

Oldenburg

- never did serious math
- advised on math questions by John Collins
- in 1676 Collins showed Leibniz a copy of Newton's De Analysi from which he made copious notes
  - opens himself to suspicion that he stole from Newton

Anagrams

6accd(ae) 13eff 7i 3l etc ...

unscrambled gives

"Data aequatione quotcunque ..."

or

"Given an equation involving any number of fluent quantities, to find the fluxions and conversely"

- This anagram

was in a letter from Newton to Leibniz on infinite series

- anagrams were used to establish priority of descovery without revealing what had been found

- Galileo had used an anagram to report that Saturn was a triple planet
  - looked like $\circ O \circ$ due to poor telescope

— 1716 – Leibniz died
  - unlike Newton, he was buried in solitude

— For more than a century

English math use $\dot{y}$, $\ddot{y}$

Continental " " $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$

to the detriment of both

HW p 405 ; 1, 4, 5, 6, 8, 10

History of Mathematics – Dr. T. J. Osler
Last Lecture Outline
Math from Newton to Today

1. Isaac Newton 1642-1727	Invents calculus – General laws of motion
Robert Hooke 1635-1703	Challenges Newton – Newton stops writing
Edmund Halley 1656- 1742	Encourages Newton to resume publishing
Gottfried Leibniz 1646-1716	Invents calculus independently
2. Bishop George Berkley 1685-1753	Points out the lack of rigor in calculus
3. Leonhard Euler 1707-1783	Most prolific mathematician
4. Joseph Louis Lagrange 1736-1813	Celestial mechanics, solution of algebraic equations.
5. Pierre Simon Laplace 1749-1827	Celestial mechanics without vectors. Stability of the solar system
6. Carl Friedrich Gauss 1777-1855	Computes orbit of asteroid Ceres
7. Jean Baptiste Joseph Fourier 1768-1830	Invents Fourier series – renews search for rigorous foundations of calculus
8. Augustine Louis Cauchy 1789-1857	Rigorous foundations for calculus
9. Evariste Galois 1811-1832	Invents modern algebra
10. Bernhard Riemann 1826-1866	Non-Euclidean geometry and the Riemann hypothesis
11. Jules Henri Poincare 1854-1912	Finds chaos in the "Three body problem"
12. Albert Einstein 1879-1955	Special and general relativity
13. Godfrey Harold Hardy 1877-1947	Major work on series and integrals
Srinivasa Ramanujan 1887-1920	Self taught genius- new series for pi
14. John Von Neumann 1903-1957	Game theory – computer visionary
15. Benoit Mandelbrot 1924-	Fractal geometry
Mitchell Jay Feigenbaum 1944-	Feigenbaum constant